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Helv. Physica Acta · Vol. 13 (3), 1940, p. 204. - W. Pauli

Invariance of Dirac's wave eqn. for similarly transformations of the line-element in the case of zero rest mass.

§1. Introduction: In general relativity the line element

$$ds^2 = g_{ik}\, dx^i dx^k \qquad (1)$$

has an absolute significance; its numerical value is determined by natural measuring rods & clocks. Therefore in this theory the natural laws are not invariant for a transformation of the line-element according to

$$ds' = f(x) \cdot ds\,; \quad g'_{ik} = f^2(x)\, g_{ik} \qquad (2)$$

in so far as $f(x)$ is allowed to be a completely arbitrary function of the coordinates. The condition of a general covariance of natural laws for such a transformation [1] which relates only to ratios of g_{ik} & gives only invariance of the light cone $ds^2 = 0$ will be called a "conformal transformation". One could hardly expect invariance against such transformations of wave eqns. for elementary particles of rest mass $\neq 0$ since according to the quantum theory the product of reciprocal of rest mass with $\hbar/c$ gives an absolute length.

On the other hand it is well known that Maxwell's eqns are invariant against conformal transformations (2). Further, the ordinary scalar wave eqn

$$\frac{\partial}{\partial x^i}\left(\sqrt{|g|}\, g^{ik} \frac{\partial u}{\partial x^k}\right) = 0 \qquad (g = \mathrm{Det}\,\|g_{ik}\|)$$

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(1) Such a condn has, as is well known, played a role in Weyl's theory where the transf. (2) accg. to

$$\varphi_i' = \varphi_i - 2\,\frac{1}{f}\,\frac{\partial f}{\partial x^i}$$

is connected with the gauge-transf. Also Einstein (Berl. Ber. 1921, p. 261) has discussed general conformal [illegible] of hydrostatics.

which in the quantum theory represents particles of zero-rest mass & zero spin has no invariance against conformation transformations.

It is therefore interesting to investigate how Dirac's wave eqn corresponding to particles of spin ½ behaves for conformal transformations (2). It is shown that in the special case of zero restmass of the particle, the Dirac eqn, analogous to the Maxwell eqn, has the property of conformal invariance wherein the wave function is so transformed that the components of vector density of the vector current are invariant.

§ 2. Mathematical procedure.

We follow here the representation of Dirac's wave eqn in the gravitation field as given by Schrodinger (Berl. Ber. 1932, p. 105, cf. also Bargmann p. 346, also Pauli. Ann. d. Phys 18, 337, 1933). The four coordinate x^k shall be real & we follow the convention that in the quadratic of the line element we shall have for the eigenvalues of the quadratic (1) a positive sign in the time-dimension & a negative sign for the three space dimensions. So that in the special relativity case (1) goes into $(dx^4)^2 - d\vec{x}^2$.

The four-rowed quadratic matrices γ_i then satisfy the relations

$$\gamma_i \gamma_k + \gamma_k \gamma_i = 2 g_{ik} \qquad (3)$$

The Dirac-eqn in a gravitational field then reads

$$\gamma^k \left(\frac{\partial}{\partial x^k} - \Gamma_k \right) \psi = i\mu \psi \qquad (4)$$

where the raising of the index in γ^k is, as usual, done with the aid of g^{ik}. The matrices Γ_k have to satisfy the relations

$$\Gamma_k \gamma_i - \gamma_i \Gamma_k = \frac{\partial \gamma_i}{\partial x^k} - \Gamma_{ik}^r \gamma_r \qquad (5)$$

where Γ^r_{ik} signifies the ordinary Christoffel symbol

$$\Gamma^r_{ik} = \frac{1}{2} g^{rs} \left(\frac{\partial g_{is}}{\partial x^k} + \frac{\partial g_{ks}}{\partial x^i} - \frac{\partial g_{ik}}{\partial x^s} \right) \qquad (6)$$

From the relations (5) the Spurs of the matrices Γ_k can be allowed to be completely free, and we shall postulate only that they satisfy

$$\text{Spur} \left(\Gamma_k + \Gamma_k^+ \right) = \frac{\partial F}{\partial x^k} \qquad (7)$$

Here + in what follows we shall understand by + always the matrix which is Hermitian-conjugate to the given matrix. The imaginary part of the Spur Γ_k can, according to Schrödinger, be taken proportional to the four-potential of the external electromagnetic field.

The given eqns are not only invariant for coordinate-transformations of the x^k, but also for arbitrary S-transformations $\psi' = S\psi$ of the spin space (without limitation on the determinant of S), wherein besides such coordinate transformations which leave the g_{ik} fixed, correspond to an S-transformation of the ψ for fixed r_i.

The wave eqn (4) acquires a physical content first of all by specifying the four ~~coarse~~ current or ~~beta~~ physical magnitudes built bilinearly from ψ^* & ψ analogously. If β be a matrix which satisfy the relations

$$r_i^+ = \beta r_i \beta^{-1}, \quad \beta^+ = \beta \qquad (8)$$

(βr_i - hermitian), then the real vector density

$$\mathfrak{f}^k = S^k \sqrt{|g|} = \psi^* \beta r^k \sqrt{|g|}\, \psi \qquad (9)$$

satisfies the continuity eqn

$$\frac{\partial \mathfrak{f}^k}{\partial x^k} = 0 \qquad (10)$$

when the magnitude F defined by (7) is connected with the determinant of β according to

$$F = -\log \mathrm{Det}\cdot\beta \qquad (11)$$

one obtains this relation from

$$\frac{\partial \beta}{\partial x^k} + \Gamma_k^{+}\beta + \beta\Gamma_k = 0$$

by multiplication with β^{-1} and building of spurs. If γ^{0k} be Dirac matrices which satisfy the eqns $\gamma_i^0\gamma_k^0 + \gamma_k^0\gamma_i^0 = 2\delta_{ik}$, then in the limiting case of special relativity our γ_i go over (according to our convention re. sign of dS^2) into

$$\gamma_\alpha = i\gamma_\alpha^0 \text{ for } \alpha = 1,2,3\,; \quad \text{and } \gamma_4 = \gamma_4^0$$

so that according to (8) & (11) we can put

$$\beta = \gamma_4^0$$

we can now determine the behaviour of the wave eqn (4) for the conformal transformation (2). From (3) we get

$$\gamma_i' = f\gamma_i \qquad (12)$$

while for the Christoffel-symbol (6) we have

$$\Gamma'^{r}_{ik} = \Gamma^r_{ik} + \frac{1}{f}\left(\delta_i^r\frac{\partial f}{\partial x^k} + \delta_k^r\frac{\partial f}{\partial x^i} - g_{ik}g^{rs}\frac{\partial f}{\partial x^s}\right) \qquad (13)$$

The solution of the eqn (5) for the Γ_k' then give, with the abbreviation

$$\gamma_{[ik]} = \tfrac{1}{2}(\gamma_i\gamma_k - \gamma_k\gamma_i),$$

$$\Gamma_k' = \Gamma_k + \tfrac{1}{2}\cdot\frac{1}{f}\frac{\partial f}{\partial x^s}g^{rs}\gamma_{[rk]} + k\cdot\frac{1}{f}\cdot\frac{\partial f}{\partial x^k}. \qquad (14)$$

where the arbitrary numerical factor k determines the Spur of Γ_k' according to

$$Spur\ \Gamma_k' = Spur\ \Gamma_k + 4k \frac{1}{f} \frac{\partial f}{\partial x^k}$$

so that according to (7)

$$F' = F + 4(k+k^*) \log f \tag{15}$$

Noticing that $\gamma^k \gamma_{[\tau k]} = -3\gamma_\tau$, it follows from (14)

$$\gamma'^k \Gamma_k' = \frac{1}{f}\left[\gamma^k \Gamma_k + (k - 3/2) \frac{1}{f} \frac{\partial f}{\partial x^\tau} \gamma^\tau\right]$$

From $$\gamma'^k \left(\frac{\partial}{\partial x^k} - \Gamma_k'\right) \psi' = i\mu\psi'$$

follows accordingly with

$$\psi' = \Omega f^{-3/2 + k} \tag{16}$$

$$\gamma^k \left(\frac{\partial}{\partial x^k} - \Gamma_k\right) \Omega = i\mu f \Omega \tag{17}$$

From (11) and (15) follows, in consonance with (8)

$$\beta' = \beta f^{-(k+k')} \tag{18}$$

accordingly on account of $\sqrt{|g'|} = \sqrt{|g|}\, f^4$

$$\psi^* \beta' \gamma'^k \sqrt{|g'|}\, \psi' = \Omega^* \beta \gamma^k \sqrt{|g|}\, \Omega \tag{19}$$

We notice that the mass term in (17) hinders, as was to be expected, the conformal invariance of the system of eqns. In the special case $\mu = 0$, we content, according to (16) & (17)

$$\Omega = \psi, \quad \psi' = \psi f^{-3/2+k} \qquad (\text{for } \mu = 0) \qquad (20)$$

and from (9) & (19) also follows for the vector density the invariance indicated in the introduction viz

$$f'^k = f^k \quad \text{for } \mu = 0 \qquad (21)$$

In this connection it can further be indicated that for every conformally invariant theory the spur $\sum_i T_{ii}$ of the energy tensor T_{ik} should vanish as one could recognise from the action principle $\delta \int L \, d^4x = 0$, and the conformal invariance of the Lagrange function. For non-vanishing rest mass this condition is never satisfied.

Internal Ballistics – H.M.S. Publication, 1951.

Chap I – Introduction: Propellants — composition of gun powder – powder test by Bourne – measurements of the ballistics of the powder – Robins' ballistic pendulum to determine muzzle velocity & later Hutton – measurement of pressure of propellant gases & setting relation between density & pressure by Count Rumford – Relation between pressure & shot travel using Rumford's p-ρ relation assuming all-burnt before shot started & calculation of muzzle velocity by integration. No ideas regarding temperature of the gases – Piobert's laws of burning in 11 layers & approximate ~~relation between~~ solution to problem of motion of gases in the bore also treated by Lagrange. Approximate relation between pressures on the breech & the base of the shot – Rodman's indentation pressure gauge for measurement of pressure of gases, measurement of max. $\underline{p}$ & $\underline{p}$-ρ relation in closed vessel, problem of reducing max. $\underline{p}$ without reducing muzzle $\underline{v}$, suggestion as to proper shape for gun powder grains viz compression into perforated discs & "prismatic powders" – Noble's crusher gauge & Nobel – Abel law relating to p & ρ at constant V – Resal's energy equation, and this law with Berthelot's work on heats of formation & reaction placed the subject on a sound thermodynamic basis.

Later developments

P.R.S. (A) 257 (1960), 32-43. — Dirac — A reformulation of the Born-Infeld Electrodynamics.

Summary — A_μ, — $f_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$

$$\mathcal{L} = -\tfrac{1}{4} f_{\mu\nu} f^{\mu\nu} \quad \text{---(1)}$$

first replaced by Born by $\mathcal{L} = -\{1 + \tfrac{1}{2} f_{\mu\nu} f^{\mu\nu}\}^{1/2}$ — (2) to preserve relativistic invariance $\mathcal{L}$ must involve $f_{\mu\nu}$ only through the two invariants $f_{\mu\nu} f^{\mu\nu}$ and $f_{\mu\nu} f^{*\mu\nu}$ (f^* being dual of f)

Later Born and Infeld favoured

$$\mathcal{L} = -\left\{1 + \tfrac{1}{2} f_{\mu\nu} f^{\mu\nu} - \left(\tfrac{1}{4} f_{\mu\nu} f^{*\mu\nu}\right)^2\right\}^{1/2} \quad \text{--- (3)}$$

Born & Infeld showed that (2) or (3) makes the total energy round a singularity finite, and also makes the potentials and their first derivatives at the singularity finite. The first derivatives are, however, discontinuous at a singularity.

In order to deal with quantisation, one must put the theory into the Hamiltonian form, which requires that one shall have an action principle for the eqns of motion. It must be a comprehensive action principle giving all the eqns of motion, those of the particles as well as those of the field. Such a comprehensive action principle has not been given, and forms the first problem dealt with here.

With the help of this principle a precise treatment of the eqns of motion of a particle becomes possible and is given in § 3. In § 4 it is shown how the ordinary Lorentz eqns of motion appear as an approx. The work up to this stage is carried through for an arbitrary $\mathcal{L}$ satisfying the requirements of relativistic and gauge invariance, and giving singularities of a similar kind to those of (2) and (3).

The later work is concerned with the passage to the Hamiltonian. For this one must work with a particular $\mathcal{L}$. There are not many choices for $\mathcal{L}$ that lead to tractable equations. The simplest one is (3), the one adopted by Born and Infeld, and this one will be used here.

§ 2. The comprehensive action principle.

It will be sufficient to take into account just one particle. Let e be the charge of the particle, and M its bare rest-mass that is not of electromagnetic nature. We may have $M = 0$, which is probably the case for an electron.

Let z^μ denote the coordinates of a point on the world-line of the particle. We introduce some arbitrary parameter τ to specify a point on the world-line, so the z^μ becomes functions of τ as these functions serve to define the world line.

We now take the action to be the sum of three terms

$$I = I_1 + I_2 + I_3,$$

where $$I_1 = \int \mathcal{L}\, d^4x \quad \text{---(4)}$$

$$I_2 = -e \int A_\mu(z) (dz^\mu/d\tau)\, d\tau \quad \text{---(5)}$$

In I_3, the integration taken over the singularity gives only a finite contribution since d is (3), unlike (2)
the Maxwell (1) for which contribution would be ∞. In I_2 appear $A_\mu(z)$ of the Born-Infeld type singularity δ not the Maxwell type
For the variation procedure, we must choose some ind. variables q, make arbitrary δq and work out the resulting
δI. The q's must be so chosen than δI depends linearly on the δq's.

One might think of taking as q's the $A_\mu(x)$ and $z^\mu(\tau)$. But then δI would not depend linearly on the δA_μ and δz^μ. One can see this by taking the special case of $\delta A_\mu(x) = 0$. Then the $\delta A_\mu(z)$ which occurs in δI_2 does not depend linearly on the δz^μ on account of the derivatives of $A_\mu(x)$ being discontinuous at $x = z$.

We can avoid the difficulty in the following way. We refer our action integral to a general set of curvilinear coords x^μ. We now keep $z^\mu(\tau)$ const. and vary the world line by varying the coordinate system. Such a variation, combines with $\delta A^\mu(x)$ equal to a non-singular function of the x^μ, means a shifting of the world line together with the singularity in the field around it, and gives a value of δI depending linearly on the parameters that determine the variation of the coordinate system and on the $\delta A^\mu(x)$. We get in this way a variation that is physically the most general permissible one and is adequate for fixing all the eqns of motion, without any variation being made in the $z^\mu(\tau)$. One could, for example, set up a system of coords. such that the particle is at the origin $z^1 = z^2 = z^3 = 0$, and it would remain at the origin under the variation.

The variation in the coord. system involves a variation in the $g_{\mu\nu}$. We shall not consider a general variation in the $g_{\mu\nu}$ as this would bring in the gravitational field. We shall restrict ourselves to $g_{\mu\nu}$'s that apply to a flat space, and consider only the variations $\delta g_{\mu\nu}$ that preserve the flatness.

A convenient way to take these restrictions into account is to introduce a set of orthogonal rectilinear coords y_K, which are kept fixed. The curvilinear coord. system can then be specified by the functions y_K and its variation by the quantities $\delta y_K(x)$. We have $g_{\mu\nu} = y_{K,\mu}\, y_{K,\nu}$, the summation over K being understood to be combined with the + sign for $K = 0$, and − sign for $K = 1, 2, 3$. Thus

$$\delta g_{\mu\nu} = y_{K,\mu}\, \delta y_{K,\nu} + y_{K,\nu}\, \delta y_{K,\mu} \qquad \cdots (7)$$

and we get δI depending linearly on the δA^μ and δy_K for all x

The type of singularity that occurs with the Born-Infeld electrodynamics is

$$A^\mu(x) = a\, r\, dz^\mu/ds + \text{less singular terms} \qquad \cdots (8),$$

where a is a const., and r is the l.v. dist. of the field point x from the world line of the particle. It involves the contravariant potentials A^μ, and so gets preserved by a variation consisting of a non-singular $\delta y_K(x)$ and a non-singular $\delta A^\mu(x)$. Such a variation leads to a $\delta A_\mu(x)$ with a singularity

of the strength of a discontinuity in its first derivatives. In spite of this singularity, δI can be expressed linearly in terms of δA_μ and δy_κ, and this expression is more convenient to work with than one in terms of δA_μ δA^μ and δy_κ.

The action density $\mathcal{L}$ in (4) is a function only of the $f_{\mu\nu}$ and $g_{\mu\nu}$. Thus

$$\delta \mathcal{L} = \tfrac{1}{2} p^{\mu\nu} \delta f_{\mu\nu} - \tfrac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \qquad \ldots (9)$$

where,

$$p^{\mu\nu} = -p^{\nu\mu} = 2 \partial \mathcal{L} / \partial f_{\mu\nu} \qquad \ldots (10)$$

$$T^{\mu\nu} = T^{\nu\mu} = -2 \partial \mathcal{L} / \partial g_{\mu\nu} \qquad \ldots (11)$$

For a relativistic theory $\mathcal{L}$ must be a scalar density. This condition leads to

$$T^\nu_\sigma = p^{\nu\rho} f_{\sigma\rho} - g^\nu_\sigma \mathcal{L} \qquad \ldots (12)$$

as follows from a small modification in formula (4·5) of Born-Infeld's paper of 1934 (P.R.S. 144, 425), or from an application of the general result of Rosenfeld's formula (29) [Acad. Roy. Belg., 18, fasc. 6].

§ 3. The exact equations of motion.

We shall proceed with the variation of the action $I_1 + I_2 + I_3$. We have, using (9) and (7)

$$\delta I_1 = -\int \left\{ p^{\mu\nu} \delta A_{\mu,\nu} + y_{K,\mu} T^{\mu\nu} \delta y_{K,\nu} \right\} d^4x \qquad (13)$$

$$= \int \left\{ p^{\mu\nu}{}_{,\nu} \delta A_\mu + \left(y_{K,\mu} T^{\mu\nu} \right)_{,\nu} \delta y_K \right\} d^4x \qquad (14)$$

Again

$$\delta I_2 = -e \int \delta A_\mu(z) \left(dz^\mu / d\tau \right) d\tau$$

$$= -e \int v^\mu ds \int \delta_4 (x-z) \delta A_\mu(x) \, d^4x \qquad \ldots (15)$$

where $v^\mu = dz^\mu/ds$. Note that the 4-dimensional δ-function $\delta_4(x-z)$ is a function of s through z being a function of s. Finally

$$\delta I_3 = -M \int \delta \left(g_{\mu\nu} \frac{dz^\mu}{d\tau} \cdot \frac{dz^\nu}{d\tau} \right) d\tau = -\tfrac{1}{2} M \int \left(g_{\rho\sigma} \frac{dz^\rho}{d\tau} \frac{dz^\sigma}{d\tau} \right)^{-1/2} \frac{dz^\mu}{d\tau} \frac{dz^\nu}{d\tau} \delta g_{\mu\nu} \, d\tau$$

$$= -M \int v^\mu v^\mu y_{K,\mu} \delta y_{K,\nu} \, ds = -M \int y_{K,\mu} v^\mu \left(d\delta y_K / ds \right) ds$$

$$= M \int \left\{ d \left(y_{K,\mu} v^\mu \right) / ds \right\} \delta y_K \, ds \qquad \ldots (16)$$

After making the variation, we have passed from τ to s as the parameter specifying the point on the world line.

Equating to zero the coefficient of δA_μ in (14) + (15) + (16), we get

$$p^{\mu\rho}{}_{,\nu} = e \int v^\mu \delta_4 (x-z)\, ds \qquad \ldots\ldots (17)$$

These are the field eqns of Born and Infeld generalised to take into account a point charge at $x = z$.

Equating to zero the coefficient of δy_K, we get

$$-\left(y_{K,\mu} T^{\mu\nu}\right)_{,\nu} = M \int \left\{ d(y_{K,\mu} v^\mu)/ds \right\} \delta_4 (x-z)\, ds \qquad \ldots (18)$$

Now,
$$y_{K,\sigma} \left(y_{K,\mu} T^{\mu\nu}\right)_{,\nu} = \left(y_{K,\sigma} y_{K,\mu} T^{\mu\nu}\right)_{,\nu} - y_{K,\sigma\nu} y_{K,\mu} T^{\mu\nu}$$

$$= T^\nu_{\sigma,\nu} - \tfrac{1}{2} g_{\mu\nu,\sigma} T^{\mu\nu}$$

$$= \left(p^{\nu\rho} f_{\sigma\rho}\right)_{,\nu} - \mathcal{L}_\sigma - \tfrac{1}{2} g_{\mu\nu,\sigma} T^{\mu\nu}, \text{ from (12).}$$

With the help of (10) and (11), and also $f_{\rho\sigma,\nu} + f_{\sigma\nu,\rho} + f_{\nu\rho,\sigma} = 0$, this becomes

$$y_{K,\sigma} \left(y_{K,\mu} T^{\mu\nu}\right)_{,\nu} = p^{\nu\rho}{}_{,\nu} f_{\sigma\rho} \qquad \ldots\ldots\ldots (19)$$

So (18) multiplied by $y_{K,\sigma}$ becomes, with the help of (17)

$$e f_{\sigma\rho} v^\rho = M y_{K,\sigma} d\left(y_{K,\mu} v^\mu\right)/ds$$

$$= M g_{\sigma\rho} \left(dv^\rho/ds + \Gamma^\rho_{\mu\nu} v^\mu v^\nu\right) \qquad \ldots (20)$$

where $\Gamma^\rho_{\mu\nu}$ is the Christoffel symbol

We have here an equation of motion for the particle. It is like Lorentz's equation with the field $f_{\sigma\rho}$ acting on the charge e. However, the L.H.S. is not well defined because $f_{\sigma\rho}$ is discontinuous at $x = z$. The equation therefore needs amendment.

The error arose in the passage from (13) to (14) as the partial integration is not valid with the domain of integration extended over the singular line. To correct the work, we must cut off the domain of integration at a small distance ϵ from the singular line. The error introduced by the cut-off tends to zero as $\epsilon \to 0$, as the integrand in (13) has no singular singularity of the nature of a δ-function. The cut-off brings an exact term into (14), of the form of a surface integral over a 3-dimensional tube-like surface surrounding the singular line, namely

$$\int \left\{ p^{\mu\nu} \delta A_\mu + y_{K,\mu} T^{\mu\nu} \delta y_K \right\} d S_\nu \qquad \ldots (21)$$

Let us attach three parameters to each point of the surface viz s, u_1, u_2 with s having the same value as for a point on the world-line at a distance of order ϵ. Then

$$dS_\nu = \epsilon_{\nu\alpha\beta\gamma} \frac{\partial x_\alpha}{\partial s} \frac{\partial x_\beta}{\partial u_1} \frac{\partial x^\gamma}{\partial u_2} ds\, du_1 du_2 = ds\, d\sigma_\nu \quad \ldots (22)$$

say, with $d\sigma_\nu = \epsilon_{\nu\alpha\beta\gamma} \frac{\partial x_\alpha}{\partial s} \frac{\partial x_\beta}{\partial u_1} \frac{\partial x^\gamma}{\partial u_2}$. We may neglect the variation of δA_μ, δy_K through distances of order ϵ, and so we can replace these quantities in (21) by their values at the point s on the singular line and get

$$\int ds \left\{ \delta A_\mu \int p^{\mu\nu} d\sigma_\nu + \delta y_K \int y_{K,\mu} T^{\mu\nu} d\sigma_\nu \right\} \quad \ldots (23)$$

The expression (23) must be added to the previous expression for δI, namely (14) + (15) + (16). Equating to zero the total coefficient of $\delta A_\mu(s)$, we get

$$\int p^{\mu\nu} d\sigma_\nu = e v^\mu \quad \ldots (24)$$

This result is in agreement with (17) and is included in it.

Equating to zero the total coefficient of $\delta y_K(s)$, we get

$$-\int y_{K,\mu} T^{\mu\nu} d\sigma_\nu = M d\left(y_{K,\mu} v^\mu\right)/ds \quad \ldots (25)$$

Multiplying by $y_{K,\sigma}$, and using (12), we get, with neglect of terms of order ϵ

$$\begin{aligned} \int p^{\rho\nu} f_{\sigma\rho} d\sigma_\nu &= M y_{K,\sigma} d\left(y_{K,\mu} v^\mu\right)/ds \\ &= M g_{\sigma\rho} \left(\frac{dv_\rho}{ds} + \Gamma^\rho_{\mu\nu} v^\mu v^\nu \right) \end{aligned} \quad \ldots (26)$$

The $p^{\rho\nu}$ in the integrand here is of order ϵ^{-2}, and to this order its value is the same as it would be if the particle at the point s were moving with a uniform velocity. The $f_{\sigma\rho}$ is of order 1, so the integral itself is of order 1.

We can express $f_{\sigma\rho}$ as the sum of two parts

$$f_{\sigma\rho} = f^{\text{sing}}_{\sigma\rho} + f^{\text{rem}}_{\sigma\rho}$$

where the main part of the singularity is put into the first terms, and the remainder is continuous and has a definite value on the singular world line. We can make $f^{\text{sing}}_{\sigma\rho}$ determinate on the tube, with neglect of order ϵ, by taking its integral to be $f_{\sigma\rho}$ produced by the particle at s moving with a uniform velocity in the absence of any external field. Thus $f^{\text{rem}}_{\sigma\rho}$ is made determinate. The r.h.s. of (26) now becomes

$$\int p^{\rho\nu} f^{\text{sing}}_{\sigma\rho} d\sigma_\nu + f^{\text{rem}}_{\sigma\rho} \int p^{\rho\nu} d\sigma_\nu$$

The first term here is the same as for a particle moving with uniform velocity, and it vanishes from symmetry. So (26) becomes finally, with the help of (24),

$$e f^{rem}_{\sigma\rho} \cdot v^{\rho} = M g_{\sigma\rho} \left(dv^{\rho}/ds + \Gamma^{\rho}_{\mu\nu} v^{\mu} v^{\nu} \right) \qquad \ldots (27)$$

In this way we get an interpretation for the ambiguous term on the left-hand side of (26), and we have well defined equations of motion for the particle.

§ 4. The Lorentz approximate equations of motion.

The eqns of motion (27) are superficially like the Lorentz eqns, but differ from them in two respects. First, the mass M is only the non-electromagnetic mass. Secondly, the field f^{rem} is not at all equal to the external field. A field coming from outside and falling on the particle, gets substantially modified in passing through the region close to the singularity, on account of the non-linear character of the field eqns, and so it gives rise to a field f^{rem} at the centre very different from what the field there would be if there were no charged particle.

The calculation of f^{rem} will usually be very complicated and will make the equations (27) difficult to apply to practical examples. One can develop approximate eqns of motion not involving f^{rem}, which are applicable when the external field is weak and involves only waves of long wave length, so that the acceleration of the particle is small. The first stage of this approximation will now be worked out.

Take a length Γ which is small compared to the wave length of the incident waves and to the radius of curvature of the world line, but large compared to the classical radius of the particle. Construct a second tube around the world line, with radius Γ. Integrate eqn (25) over a small range of s of values, say Δs, so that it becomes, from (22),

$$- \int \gamma_{K,\mu} T^{\mu\nu} dS_{\nu} = \Delta s \, M \, d\left(\gamma_{K,\mu} v^{\mu} \right)/ds \qquad \ldots (28)$$

where the L.H.S. is integrated over a small length of the tube corresponding to Δs. The integrand here has its divergence vanishing everywhere except on the singular world line, as is shown by (19) and (17). We may therefore replace the surface of integration by any other surface with the same boundary, provided the region between the two surfaces does not contain any of the singular world line. Let us take the second surface to be the corresponding length Δs of the tube Γ, together with two end surfaces that connect the ends of the bit of the tube ϵ with those of the bit of the tube Γ.

The 4-th integral over the surface of the tube τ can be handled by the same method as was used for the tube ϵ (the eqns now being only approximate since we cannot make $\tau \to 0$), with the Coulomb field playing the role of $f^{sing.}$, and the external field $f^{ext.}$ playing the role of $f^{rem.}$. Bringing in the multiplying factor $y_{K,\sigma}$, we get for this term

$$\Delta s\, e\, f^{ext.}_{\rho\sigma} \cdot v^\rho .$$

We can evaluate the integral over one of the end surfaces, to the first order of accuracy in τ, by neglecting the external field, and the acc^{n} of the particle, and we then find it for it $-m\, y_{K,\mu}\, v^\mu$, where m is the mass of the field around a stationary charge e, a mass that is finite with the Born-Infeld type of field. The contribution of both end surfaces is thus $-\Delta s\, m\, d(y_{K,\mu} v^\mu)/ds$.

So eq^{n} (28), multiplied by $y_{K,\sigma}$ and divided by Δs, becomes

$$e f^{ext.}_{\sigma\rho} v^\rho = (M+m)\, y_{K,\sigma}\, d(y_{K,\mu} v^\mu)/ds$$
$$= (M+m)\, g_{\sigma\rho} \left(\frac{dv^\rho}{ds} + \Gamma^\rho_{\mu\nu} v^\mu v^\nu \right) \quad \ldots\ldots (29)$$

These are just the Lorentz eqns of motion. They have previously been obtained by other authors by other methods, for eg. Born & Infeld (1934), Frenkel (1934), Teerburg (1935) and Pryce (1936). The present work shows that they are approximate consequences of the configuration action principle. If this work were carried through with greater accuracy the Lorentz damping term would have to appear, because there is strict conservation of energy and momentum.

§ 5. The Hamiltonian in the absence of charges.

We follow the general method of passing from an action principle to the Hamiltonian formulation of the equations of motion, given by the author (1951) and (1958). We assume the surfaces $x^0 =$ const. to be space-like and deal with states on these surfaces. The quantities y_K, A_μ for all x^1, x^2, x^3 for a given x^0 are the basic dynamical co-ordinates of the theory and their derivatives w.r.t x^0 are the velocities.

We shall work with the action density proposed by Born & Infeld (1934), which is extremely simple referred to curvilinear coordinates, and is favoured for that reason. Put

$$\lambda_{\mu\nu} = g_{\mu\nu} + f_{\mu\nu} ,$$

and let the determinant of the $\lambda_{\mu\nu}$ be $-\Lambda^2$. Then the action density is $\mathcal{L} = -\Lambda$. It goes over into (3) for rectilinear coordinates.

The Lagrangian L is given by

$$I = \int L \, dx^4$$

and is therefore $$L = -\int \Lambda \, d^3x .$$

Making a general variation in the $\lambda_{\mu\nu}$, we get

$$\delta L = -\tfrac{1}{2} \int \Lambda K^{\nu\mu} \delta \lambda_{\mu\nu} \, , \quad \text{where } K^{\nu\mu} \text{ is the reciprocal matrix to } \lambda_{\mu\nu} \text{ so that} \qquad \ldots (30)$$

$$K^{\nu\mu} \lambda_{\mu\rho} = K^{\mu\nu} \lambda_{\rho\mu} = g^{\nu}_{\rho} . \qquad \ldots (31)$$

Thus if we vary the velocities $y_{K,0}$, $A_{\mu,0}$, we get from (7)

$$\delta L = \tfrac{1}{2} \int \Lambda \left\{ -(K^{0\mu} + K^{\mu 0}) \, y_{K,\mu} \, \delta y_{K,0} + (K^{0\mu} + K^{\mu 0}) \, \delta A_{\mu,0} \right\} d^3x .$$

The momenta w_K and D^{μ} conjugate to the dynamical coordinates y_K and A_{μ} respectively are given by

$$\delta L = \int \left\{ w_K \, \delta y_{K,0} + D^{\mu} \delta A_{\mu,0} \right\} d^3x$$

So, comparing coefficients, we find

$$w_K = -\tfrac{1}{2} \Lambda (K^{0\mu} + K^{\mu 0}) \, y_{K,\mu} , \qquad \ldots (32)$$

$$D^{\mu} = \tfrac{1}{2} \Lambda (K^{0\mu} - K^{\mu 0}) \qquad \ldots (33)$$

The equations (32) and (33) lead to some relations between the dynamical coordinates and the momenta, independent of the velocities. These are the primary constraints of the Hamiltonian theory. One of them is evidently

$$D^0 = 0 \qquad \ldots (34)$$

Using latin suffixes to take on the values 1, 2, 3, we have

$$w_K \, y_{K,r} = -\tfrac{1}{4} \Lambda (K^{0\mu} + K^{\mu 0})(\lambda_{r\mu} + \lambda_{\mu r})$$

$$= -\tfrac{1}{4} \Lambda (K^{0\mu} - K^{\mu 0})(\lambda_{r\mu} - \lambda_{\mu r}) , \text{ from (31).}$$

Hence $$w_K \, y_{K,r} + f_{rs} D^s = 0 \qquad \ldots (35)$$

which gives us more primary constraints. Further, from (32)

$$w_K \, w_K = \tfrac{1}{4} \Lambda^2 (K^{0\mu} + K^{\mu 0})(K^{0\nu} + K^{\nu 0}) \, q_{\mu\nu}$$

$$= \tfrac{1}{4} \Lambda^2 (K^{0\mu} + K^{\mu 0})(K^{0\nu} + K^{\nu 0}) \, \lambda_{\mu\nu}$$

and similarly, from (33)

$$g_{rs} D^r D^s = \tfrac{1}{4} \Lambda^2 (K^{0\mu} - K^{\mu 0})(K^{0\nu} - K^{\nu 0}) \lambda_{\mu\nu}$$

Adding these two equations, we get

$$w_K w_K + g_{rs} D^r D^s = \tfrac{1}{2} \Lambda^2 (K^{0\mu} K^{0\nu} + K^{\mu 0} K^{\nu 0}) \lambda_{\mu\nu}$$

$$= \Lambda^2 K^{00} \qquad \ldots\ldots (36)$$

from (31). The quantity (36) is minus the determinant of the λ_{rs}, and its expansion yields

$$\Lambda^2 K^{00} = \Gamma^2 - g_{rs} B^r B^s,$$

where $-\Gamma^2$ is the determinant of the g_{rs}, and

$$B^r = \tfrac{1}{2} \epsilon^{rst} f_{st}.$$

ϵ^{rst} being anti-symmetrical with $\epsilon^{123} = 1$. Thus (36) gives

$$w_K w_K + g_{rs} (D^r D^s + B^r B^s) - \Gamma^2 = 0 \qquad \ldots\ldots (37)$$

which is another primary constraint.

We can put (37) into an alternative form. We resolve the vector w_K into components tangential and normal to the surface $x^0 = \text{const}$. The tangential components are $w_K y_{K,r} = w_r$, say, and the normal component is $w_K y_{K,\mu} g^{\mu 0} (g^{00})^{-\frac{1}{2}} = w_{nor}$, say. Equation (35) gives the tangential components in terms of the other Hamiltonian variables, and (37) can be put into a similar form in which it gives w_{nor} in terms of the other Hamiltonian variables. We have

$$w_K w_K = (w_{nor})^2 + e^{rs} w_r w_s,$$

where e^{rs} is the reciprocal matrix to g_{rs}. So (37) and (35) lead to

$$w_{nor} + \left\{ \Gamma^2 - g_{rs} (D^r D^s + B^r B^s) - e^{rs} f_{rt} D^t f_{su} D^u \right\}^{\frac{1}{2}} = 0 \qquad \ldots\ldots (38)$$

The physical meaning of various quantities here may be noted. D^r is the p^{0r} of (10) and is the electric induction, while B^r is the magnetic induction. The value of $-w_{nor}$ given by (38) is the energy density, and the value of $-w_r$ given by (35) is the momentum density or Poynting vector.

Since the velocity $A_{0,0}$ does not occur in L, we get a Lagrangian equation of motion if we vary A_0 in L, and put the coefficient of δA_0 equal to zero. The variation gives, from (30)

$$\delta L = -\tfrac{1}{2}\int \Lambda\,(K^{0\tau} - K^{\tau 0})\,\delta A_{0,\tau}\,d^3x$$

$$= -\int D^{\tau}\,\delta A_{0,\tau}\,d^3x = \int D^{\tau}_{,\tau}\,\delta A_0\,d^3x\,,$$

so we get the equation,

$$D^{\tau}_{,\tau} = 0 \qquad \ldots\ldots\ldots\

With a charged particle present, we must add to the action the term $I_2 + I_3$ given by (5) and (6). For the Hamiltonian treatment we may conveniently take $\tau = z^0$. The z^ν are to be considered as given functions of z^0, and not dynamical coordinates. The $dx^\nu/dx^\mu \, dz^\mu/d\tau = z^\nu_{,0}$ are likewise given functions of z^0, and not dynamical velocities. The presence of the particle does not bring in any extra dynamical coordinates and velocities, and thus no extra momentum variables.

The extra Lagrangian is

$$L_2 + L_3 = -e A_\mu(z) z^\mu_{,0} - M \left\{ g_{\mu\nu}(z) z^\mu_{,0} z^\nu_{,0} \right\}^{1/2} \qquad \ldots\ldots (43)$$

Variation of the velocities $y_{\kappa,0}$, $A_{\mu,0}$ in it leads to

$$\delta(L_2 + L_3) = -M \left\{ g_{\rho\sigma} z^\rho_{,0} z^\sigma_{,0} \right\}^{-1/2} z^\mu_{,0} \, y_{\kappa,\mu}(z) \, \delta y_{\kappa,0}(z)$$

$$= -M v^\mu \int y_{\kappa,\mu}(x) \, \delta_3(x-z) \, \delta y_{\kappa,0}(x) \, d^3x .$$

The equation (33) for D^μ is therefore unchanged, while the equation (32) for W_κ gets changed to

$$W_\kappa = -\tfrac{1}{2} \Lambda \left(\kappa^{0\mu} + \kappa^{\mu 0} \right) y_{\kappa,\mu} - M v^\mu y_{\kappa,\mu} \, \delta_3(x-z) \qquad \ldots\ldots (44)$$

Variation of A_0 in (43) leads to

$$\delta(L_2 + L_3) = -e \, \delta A_0(z) = -e \int \delta_3(x-z) \, \delta A_0(x) \, d^3x ,$$

so the Lagrangian equation of motion (39) gets changed to

$$D^\nu_{,\nu} - e \, \delta_3(x-z) = 0. \qquad \ldots\ldots (45)$$

It is still a secondary constraint.

Corresponding to (35) we now get, with the notation of (41)

$$W_\tau + X_\tau = -M g_{\tau\mu} v^\mu \delta_3(x-z) \qquad \ldots (46)$$

For $M \neq 0$, the right hand side involves velocities, so the equation is not a constraint. However, (46) is really a large number of equations, three for each point x^ν, and for $x^\nu \neq z^\nu$ the right hand side vanishes and provides us with constraints. The set of constraints given in this way may conveniently be written

$$\int a^\tau \left(W_\tau + X_\tau \right) d^3x = 0, \qquad \ldots (47)$$

where the a^τ are functions of x^ν that are restricted to vanish for $x^\nu = z^\nu$. These constraints are first class,

and correspond to the possibility of one's making arbitrary changes in the 3-bound coordinates x^r leaving the point $x^r = z^r$ invariant.

Similarly, corresponding to the last of equations (41), we can deduce

$$w_{nor.} + \chi_{nor.} = -M v_{nor} \delta_3(x-z), \qquad \ldots\ldots (48)$$

where $v_{nor.}$ is the normal component of the vector v^μ. This gives us the constraints

$$\int b\,(w_{nor.} + \chi_{nor.})\, d^3x = 0, \qquad \ldots (49)$$

where b is a function of x^r restricted to vanish for $x^r = z^r$. These constraints are first class and correspond to the possibility of one's making arbitrary deformations of the surface $x^0 = \text{const}$ that leave the point $x^\mu = z^\mu$ invariant.

We need a further first-class constraint to correspond to the possibility of moving the point $x^\mu = z^\mu$ along the assigned world-line for the particle. To get this further constraint, we must eliminate the velocities from the right-hand sides of (46) and (48) at the point $x^r = z^r$. The result is

$$\int (w_{nor} + \chi_{nor})\, d^3x \int (w'_{nor} + \chi'_{nor})\, d^3x' + e^{rs}(z) \int (w_r + \chi_r)\, d^3x \int (w'_s + \chi'_s)\, d^3x' - M^2 = 0, \quad \ldots (50)$$

the domain of integration in each case being any domain enclosing once the point $x^r = z^r$.

For the case $M \neq 0$, the constraints (47), (49), (50), (45) and (34) are the only independent ones, and the Hamiltonian is a linear combination of their left-hand sides. The case $M = 0$ is quite different, and needs further investigation.

§ 7. Quantisation

We have a classical theory in the Hamiltonian form, with all the constraints first-class. To quantise such a theory, we must make the dynamical variables into operators satisfying commutation relations corresponding to the classical P.b. relations, and we must make each constraint, say $\phi_n = 0$ into a condition on the wave function

$$\phi_n \psi = 0.$$

In order that these conditions may be consistent with one another, it is necessary that the order of non-commuting factors in the ϕ's be suitably chosen, so that

$$[\phi_n, \phi_m] = \sum_K C_{nmK} \phi_K \qquad \ldots\ldots (51)$$

with the Born-Infeld electrodynamics there does not seem to be any way of arranging the ϕ's

to satisfy the conditions (51), even in the absence of charged particles. If we take the form (38) for some of the ϕ's, expressing $W_{nor.}$ in terms of other dynamical variables, we have square roots appearing, leading to hopeless complications in the commutations. One gets nearest to satisfying the conditions (51) if one works with the natural form (37) instead of (38), but it does not completely fit.

The Born-Infeld electrodynamics is completely satisfactory in the classical theory. It avoids all the troublesome infinities associated with point charges in the Maxwell theory. Also, it gives an expression for the energy density $-W_{nor.}$ that is positive definite, as is shown by (38) and (48), the quantities under the square root in (38) being also + ve definite. It follows that there cannot be any runaway solutions of the eqns of motion of a charged particle (in which the particle continually accelerates in the absence of an external field), such as occur with the Maxwell-Lorentz theory.

But the Born-Infeld electrodynamics cannot be quantized with the present rules of quantization, even in the absence of charges. To get a quantum theory from it one will need to make some fundamental ~~operation~~ alteration, probably involving the introduction of anti-commuting quantities into the description of the field and using them to avoid the square roots in the expression for the energy.

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How to get (3) from (2) in Nagell?

$$\frac{(x+i\sqrt{7})(x-i\sqrt{7})}{4} = \left(\frac{1+i\sqrt{7}}{2}\right)^y \quad ? \qquad \left(\frac{1+i\sqrt{7}}{2}\right)^4 \text{ is } 2 = \frac{1+i\sqrt{7}}{2}$$

$$4 = 1 + i\sqrt{7}$$

$$4 = 1 - i\sqrt{7} \qquad \underline{8 = 1+7 = 8}$$

ie

$$\frac{x+i\sqrt{7}}{2} = \left(\frac{1+i\sqrt{7}}{2}\right)^y$$

d

$$\frac{x-i\sqrt{7}}{2} = \left(\frac{1-i\sqrt{7}}{2}\right)^y$$

$$\left(\frac{1+i\sqrt{7}}{2}\right)^y - \left(\frac{1-i\sqrt{7}}{2}\right)^y = +i\sqrt{7}$$

Brauer & Schinzel – C.R. 242, 1780-81

$2^n - 1 = \frac{1}{2}K(K+1)$ is equivalent to

$$(K+\tfrac{1}{2})^2 + \frac{1}{4} = \left\{\left(\tfrac{1}{2}\right)^2 + \frac{7}{4}\right\}^{n+1}$$

hence one has

$$(K+\tfrac{1}{2}+\tfrac{1}{2}i\sqrt{7})(K+\tfrac{1}{2}-\tfrac{1}{2}i\sqrt{7}) = \left(\tfrac{1}{2}+\tfrac{1}{2}i\sqrt{7}\right)^{n+1}\left(\tfrac{1}{2}-\tfrac{1}{2}i\sqrt{7}\right)^{n+1} \qquad \ldots\ldots (1)$$

The numbers $\frac{1}{2} \pm \frac{1}{2}i\sqrt{7}$ are integers of the field $K(\sqrt{-7})$ and also prime

Putting $\alpha_K = K + \frac{1}{2} + \frac{1}{2}i\sqrt{7}$, $\beta_K = K + \frac{1}{2} - \frac{1}{2}i\sqrt{7}$, $\alpha_K - \beta_K = i\sqrt{7}$, $\alpha_K \beta_K = 2^{n+1}$

from property of fields $K(\sqrt{-7})$, we have from (1), $\alpha_K = \pm \alpha_0^{n+1}$ & $\beta_K = \pm \beta_0^{n+1}$ or $\alpha_K = \pm \beta_0^{n+1}$ & $\beta_K = \pm \alpha_0^{n+1}$

& since $\alpha_K - \beta_K = i\sqrt{7}$, we find $i\sqrt{7} = \pm \alpha_0^{n+1} \mp \beta_0^{n+1}$ or $i\sqrt{7} = \pm \beta_0^{n+1} \mp \alpha_0^{n+1}$

Putting $u_K = \frac{1}{i\sqrt{7}}(\alpha_0^K - \beta_0^K)$ for $K = 1, 2, \ldots$, we find

$$|u_{n+1}| = 1 \qquad \ldots\ldots\ldots\ldots (2)$$

we have $u_1 = u_2 = 1$, and as remarked by Watkins, $u_{K+1} = u_K - 2u_{K-1}$ for $K = 2, 3, \ldots$. The numbers u_K $(K = 1, 2, \ldots)$ are rational integers

for k, l integers. The number

$$\frac{u_{kl}}{u_k} = \sum_{j=1}^{l-1} \alpha_0^{k(l-j)} \beta_0^{kj}$$

is integral in $K(\sqrt{-7})$ and hence $u_k \mid u_{kl}$ for k & l natural $\ldots\ldots (3)$

Ng Nagell's proof of $x^2+7=2^n$ in Ark. f. Math, 4, No. 13, p. 185 — Theorem set as a problem in his Introduction to Number Theory, N.Y., 1951 (Problem 165, p. 272). — Proof of 1948 which is quite elementary.

The problem consists in determining all the $+^{ve}$ integers $x \neq 0$, y such that

$$\frac{1}{4}(x^2+7) = 2^y \quad \ldots (1)$$

It is evident that difference of two integral squares u^2 and v^2 is equal to 7 only for $u^2=16$ and $v^2=9$. Hence we conclude that the exponent y in (1) can be even only for $y=2$, and $x=3$. Thus we [illegible], so that y is odd and ≥ 3.

Passing to the quadratic field $K(\sqrt{-7})$, in which factorisation is unique

$$\frac{x \pm \sqrt{-7}}{2} = \left(\frac{1 \pm \sqrt{-7}}{2}\right)^y \quad \ldots (2)$$

whence

$$\left(\frac{1+\sqrt{-7}}{2}\right)^y - \left(\frac{1-\sqrt{-7}}{2}\right)^y = \pm\sqrt{-7} \quad \ldots (3)$$

[Proof that lower sign must be taken in (3)]. (3) can be written as

$$-2^{y-1} = \binom{y}{1} - \binom{y}{3}7 + \binom{y}{5}7^2 - \cdots \pm \binom{y}{y}7^{\frac{1}{2}(y-1)} \quad \ldots (4)$$

This eqn implies the congruence, $-2^{y-1} \equiv y \pmod 7$, which has the solutions $y \equiv 3, 5, 13 \pmod{42}$

(i) for $y \equiv 3 \pmod{42}$, putting $y-3 = 7^z \cdot 6 \cdot h$, where h is an integer not divisible by 7. using (4) leads to $y-3 \equiv 0 \pmod{7^{z+1}}$ contrary to hypothesis of y. Thus only possibility is $y=3$ corresponding to $x=5$

(ii) for $y \equiv 5 \pmod{42}$, putting $y-5 = 7^z \cdot 6 \cdot h$ [h same as in (i) not divisible by 7) & using (4) leads to $y-5 \equiv 0 \pmod{7^{z+1}}$ contrary to hypothesis on y. Thus only possibility is $y=5$ corresponding to $x=11$

(iii) for $y \equiv 13 \pmod{42}$, putting $y-13 = 7^z \cdot 6 \cdot h$, we can get the congruence

$$-4096 \equiv y - 4109 \pmod{7^{z+1}}$$

which is contrary to hypothesis on y. Thus only possibility is $y=13$, corresponding to $x=181$

Brousseau & Schinzel — CR. Acad, 242, pp. 1780–81.

Theorem: The Mersenne numbers $M_n = 2^n - 1$ (where $n = 1, 2, \ldots$) which are at the same time triangular $t_K = \frac{1}{2}K(K+1)$ are only the numbers $M_1 = t_1$, $M_2 = t_2$, $M_4 = t_5$ and $M_{12} = t_{90}$

Demonstration based on the field $K[\sqrt{-7}]$ as in Nagell, and quite complicated proof

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Same authors — Acad. Pol. V. 8/1960, pp. 311–31[illegible] — on the eqn $2^n - D = y^2$ — fewer solutions — Ref. given to above article,

It is further shown that the numbers u_n $(n=1,2,\cdots)$ form an infinite periodic set mod 64, from which one gets

$$n+1 \equiv 13 \pmod{16} \text{ and } u_{n+1} = -1 \quad \cdots (4)$$

Next one considers $v_k = u_{16k+13}$, and $w_k = u_{3k+1}$ $(k=0,1,2,\cdots)$

and using these the theorem is proved. — Quite complicated.

Mordell's proof in Ark. f. Mat. (1962) Bd. 5, h. 455–60:

Takes $2^n = x^2+7$.

(1) for n even — this occurs only when $n=4$, $(2^{\frac{1}{2}n}+x)(2^{\frac{1}{2}n}-x)=7$ & so $2^{\frac{1}{2}n}+x=7$ & $2^{\frac{1}{2}n}-x=1$

$2^{\frac{1}{2}n}=4$, $n=4$, $x=3$. This is also the only soln for which $x \equiv 0 \pmod 3$. For, then, & all to mod 3,

$$2^n - 1 \equiv 0 \quad \text{or} \quad (-1+3)^n - 1 \equiv 0, \quad (-1)^n - 1 \equiv 0$$ & so n is even.

(2) for n odd & $x \not\equiv 0 \pmod 3$ — Corresponding to the cases $x = 3m, 3m+1, 3m+2$, the respective eqns are

$$y^3 - 7 = x^2 \quad \text{——— (1)}$$
$$2y^3 - 7 = x^2 \quad \text{——— (2)}$$
$$4y^3 - 7 = x^2, \quad \text{——— (3)}$$

where $y = 2^n$.

(3) becomes Nagell's eqn when $x \to 2x+1$. Since x is odd. Proof given using field $R(\sqrt{-7})$.

For eqns (1) & (2), the field $R(\sqrt{-7})$ does not seem useful. Use is made of Størmer's

& proofs in cases of (1) & (2) are quite complicated

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by showing that the eqn $2^x - 1 = \frac{1}{2}y(y+1)$ is equivalent to $2^{x+3} - 7 = (2y+1)^2$.

There is also a paper by Nagell in the same issue of Arkiv. pp. 267-86 about the "search of two integral squares in certain quadratic fields" easily within of quadratic residues etc.

10/8/66.

Rev. Mod. Phys - January 66.

p. 36 - Complex coordinates and quantum mechanics by F. Strocchi - Quite elementary.

Ibid - April 66

p. 215 - Dynamical Symmetry in Particle Physics by A. Pais - Colossal amount of Group Theory

pp. 330 & 346 - Group Theory of the Hydrogen Atom by M. Bander and C. Itzykson I & II - Complete Greek to me

Annales de Physique, Vol. 1966, p. 271 - An account of the work of Tomonaga, Schwinger and Feynman to whom the Nobel Prize in Physics for 1965 has been given - Work in quantum electrodynamics - Solving of the divergences, renormalisation etc. - m_0 & e_0 are fictitious & in the calculations replacing these by $m_0 + \delta m$ and $e_0 + \delta e$, which alone have physical significance as given by their values m and e

25/8/66 - Scientific American, May, 1963 - First article by Dirac on the Physicists picture of Nature. Quite a beautiful - Special mention about "mistake" in renormalisation theories - Also some of class one & class two difficulties - Lines of future development.

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[Diad = Sciences & Humanities
Triad = Greek Triad of logics, ethics & aesthetics"]

22/4/67 (1) Sc. American – July 66, p. 38 – Modern Cryptology – How secret information is stolen by the making and breaking of ciphers, with the aid of electronics & mathematics. – Case of two Russian agents in 1961 arrested for having their key numbers listed on scrolls about the size of cigarette butt, hidden inside a cigarette lighter – The "one time system" not in universal use.

(2) Polarised accelerator targets – Crystals in which the nuclear particles are all spinning in the same direction are bombarded with high energy particles to measure how much the nuclear forces depend on the spin of the nuclear particles.

(3) Short term memory. (4) Math. Games, p. 108).

Scientific American, Sept 1966 – (1) All articles about computers – p. 74 on Computer logic & memory – Modern Computer "hardware" works with the binary logic of 0 and 1.

(2) Math. Games, p. 264.

– do – October, 1966. – (1) The culture of poverty – Membership in a group that has been poor for generations constitutes belonging to a separate culture. A study of Puerto Ricans in both Puerto Rico, and New York indicate that this true.

(2) The origins of the Copernican Revolution, p. 88.

(3) Mathematical Games, p. 114 by Martin Gardner

"Shuffling is the only thing which Nature cannot undo" – Boltzmann

The problem is on the shuffling of cards (and other apparently random

– do – Nov. 66 – (1) p. 37, Technology in China – Japanese analyst estimates that a breakthrough & economic growth by industrial investment may be 5 or 10 years off.

(2) p. 78 – 3-D mol structure of an oxygen molecule – beautiful pictures.

(3) Math. Games, p. 138 – Is it possible to visualise a four-dimensional figure?

" do – Dec. 66 – (1) p. 40 – Problem of the quasi-stellar objects by Burbidge & Hoyle – no hypothesis about their nature is still satisfactory.

(2) p. 128; Math. Games – The multiple charms of Pascal's triangle

13/7/68 - Ref. to Sci. American — Work under C.S.I.R. Scheme (1)
Started on 3/7/68.

✓(1) Pions by R. Marshak — Jan 57

✓(2) Overthrow of parity by P. Morrison - April, 57

✓(3) Elementary particles by Gellmann etc - July, 57

✓(4) Principle of uncertainty by G. Gamow - Jan, 58

(5) Anti-matter by Burbridge & Hoyle - April, 58

(6) The atomic nucleus by Pisa Peierls - Jan, 59

✓(7) Ionising radiation by Platzman - Sept, 59

✓(8) Exclusion principle by Gamow - July, 59

(9) High energy Cosmic rays by Rossi - Nov, 59

(10) The weak interaction by Freimann - March, 59

(11) The nuclear force by R. Marshak - March, 1960

(12) Neutrino Astronomy by P. Morrison - Aug. 1962

✓(13) The muon by S. Penman - July, 1961 (The best hope to solve the mystery* of the muon seems to lie in scattering experiments.

✓(14) Gravity by Gamow - March, 1961 (Beautiful - mentions protons & neutrons)

(15) Two neutrino experiment by Lederman - March, 1963 (also talks of muon-ness & electron-ness)

(16) Hypernuclei by Telegdi - Jan, 1962

(17) Strongly interacting particles by Chew, etc - Feb, 1964.

* Talks of it as a "riddle wrapped in a mystery inside an enigma" p. 55.

Article by Raman & Bhagavantham in Nature 1932, Jan 2nd, p. 22 - Expt. proof of the spin of the photon:

The expt. was to determine the extent to which the depolarisation of Rayleigh scattering of monochromatic light is diminished when it is spectroscopically separated from the scattering of altered frequency arising from the molecular motion in a fluid ... Something about expt. devices in the cases of gases O_2, CO_2 & N_2O (under pressure) using alternatively a fine slit and a very broad slit on the spectrograph, the depolarisation of the Rayleigh scattering & of the total scattering respectively determined - A table follows giving observed values for the three gases of total & Rayleigh scattering & Calculated values on the Fresnel-Weigert

(2) theory and the values calculated from the theory of the spinning photons discussed in our paper (in Ind. J. Physics, vol 6, p. 353, Oct 31). It will be seen that the values given by the latter are strikingly supported by the exptl. results.

Nature, 1929, II Vol. 128, 1931, II, pp. 576 & 727 (referred to in the above paper) – Raman & Bhagavantam's radiogram re "Exptl. demonstration of Spin of light" – The depolarisation of the Rayleigh scattering of monochromatic light in CO_2 does not diminish to one quarter of its value when spectroscopically defended from rotational scattering, as demanded by existing theories of radiation. The actual observed diminution from 10% to 6% is quantitatively explicable, assuming that common light consists of spinning quanta possessing one Planck unit of A.M.

Ibid. p. 727 – Refers to this using liquids.

Article by Gamow on Gravity – Sc. Amer, March 1961, p. 194

(1) Galileo's expts with inclined planes & pendulums – Mention of a celebrated legend, Galileo climbed the leaning tower of Pisa and dropped a light and a heavy ball which hit the ground simultaneously to the consternation of contemporary philosophers.

(2) Newton's law of gravity – Newton born the year Galileo died – laws of motion; notions of force and inertial mass – Acc^n is directly :: ^{l} to force and inversely :: ^{l} to mass ($F = mx$) – Gravitational mass wt of a body (ie gravitational pull of the earth upon it) is strictly :: ^{l} to its inertial mass. This proportionality is far from trivial; it holds only for gravity & not for other familiar forces such as those of electricity & magnetism. Thus an $\underline{e}$ & a $\underline{p}$ would fall with equal acc^n in a gravitational field, whereas if these particles are placed in an electric field the electron is accelerated 1836 times faster. – origin of the profoundly important idea of universal gravitation (orbital motions etc) – Demonstrated by Cavendish* (using torsion balance) half a century after Newton's death, of the weak gravitational attraction between two small bodies – $F = G.(M_1 M_2)/d^2$, G the gravitational constant being a very small number. If masses are measured in gms & distances in cms, G is approx 0.00000066 – Explanation of Kepler's laws – But the nature of gravitation, and in particular the reason for the mysterious proportionality between gravitational mass and inertial mass remained completely hidden for more than 200 years.

(3) Einstein's law of Gravity: – Formulated a special theory equivalent to saying that no observations made inside an enclosed chamber can reveal the question of whether the chamber is at rest or moving

along a st: line at constant speed – Example of an observer (inside) an accelerated space ship lets fall down releasing two spheres (one of iron & one of wood) & noticing that they hit the floor simultaneously. The inside observer thinks that there exists a gravitational field inside the space ship. For an outside observer, the explan is that the 2 balls had the same vel! when released, while the ship moving upwards with acceleration will gain speed and the "floor" of the ship will meet the two spheres & hit them simultaneously – Both descriptions are correct, & the equivalence of the two points of view is the foundation of Einstein's general theory of relativity or relativistic theory of gravity – Einstein's deep insight that this Principle of Equivalence is true not only for mechanical phenomena, but is quite general and holds also for optical and other electro-magnetic phenomena – Ex. of curvature of light: A beam of light propagating with horizontal velocity across the space laboratory; explanations from points of view of observers outside and inside the Laboratory; the latter drawing a line through the flourescent spots will see a parabola bending towards the floor. Since he considers acc^{n} phenomena as being caused by gravity, he will say that a light ray is bent when propagating through a gravitational field – Einstein's conclusion that light rays from distant stars that pass close to the sun on their way to the earth should bend towards the Sun is brilliant confirmation of this theory by total eclipse of the sun in 1919 by British astronomers.

(4) Relativistic or a Merry-go-Round. – In uniform rotation round an axis, there is also accelerated motion directed away from the centre (centrifugal) and $\propto$ (necessity) to dist. from centre ($\propto \frac{1}{r^2}$ decreasing). Here also, where there is cylindrical and not spherical symmetry, the equivalence principle holds. For eg. for people in a merry-go-round with a curtain round it not aware of its rotation would imagine the centrifugal force caused by gravitating mass distributed at large distances all around the symmetry axis. For outside observers the centrifugal force is $\propto$ to the inertial mass.

Re. path of ray of light in such a gravitational field, consider path between two points A & B on the periphery of the disc. The path must be a geodesic according to laws of Optics. Suppose measurement is made by using small yardsticks and counting their number to find distance between A and B. Watching this expt. from outside, we recall that according to special theory of relativity shrinks in the direction of their motion (i.e. away from centre) and more sticks are needed than to measure the "true" distance between A and B when the platform is at rest. Hence an observer on the platform would find that a curved line is the S.D between A & B. Further, the closer a yardstick is to the centre, the less its velocity and hence the smaller its contraction. By bending the line of yardsticks towards the centre, the observer decreases the number he needs to go from A to B and the "actual" distance although somewhat longer is more than compensated for by the smaller shrinkage of the yardsticks. Thus for an observer on the platform a light ray at first goes to bend inwards at the start of its journey and then bending outwards due to the apparent

gravitational field.

Again a clock at the edge of the periphery of is with more[illegible] than one at the centre, as required by the special theory of relativity. Hence the clock as required by the special theory the outer clock will lose time w.r.t the inner one. The explanation in terms of gravitational fields (eg accn effects) is that a clock placed at a higher gravitational potential (ie in the area in which the gravitational force acts) runs slower.

Einstein argued that the same must be true of any gravitational field for eg. the one on the earth. Here the field is directed downward so that a clock at sea level runs slower than one on the top of a mountain. The slowing down applies equally well to all other physical, chemical and biological phenomena [A physicist working on the first floor of the Empire State Building will age slower than her twin sister working on the top floor!]. Stronger fields produce greater retardation in time & for eg. a clock on the surface of the Sun would run 0.0001 per cent slower than a terrestrial clock — "the gravitational red shift" was predicted by Einstein but is too small to be measured. For white dwarf stars, this shift is nearly 40 times more, and observations agree with Einstein's theory.

Astronomical evidence not so satisfactory as expts that can be performed in a terrestrial laboratory. The measurement of the extremely minute differences between different clocks at different heights in the earth's gravitational field has recently been made possible by the Mossbauer effect (Sc. Amer., April 1960). Mossbauer found a way to produce nuclear gamma rays of very pure frequency and to measure extremely small changes in their frequency. Using this effect, several workers were able to show that two nuclear "clocks" separated by only a few tens of feet in the earth's field run at measurably different rates & the difference is exactly that predicted by Einstein. Still another verification will perhaps be obtained in the future by comparing the rates of an atomic "clock" in an artificial satellite with one on the ground.

To explain these phenomena of clocks running slow, light rays bending, the st: line not being necessarily the S.D. between two points, Einstein suggested more generally that Space (more accurately space-time) is itself curved — Examples of sum of angles of a triangle being $\gtrless 180°$ and spherical, plane, and saddle like surfaces (Δ formed by geodesics), and curved, defining surfaces according to these relations being +ve curved, flat or −ve curved. For eg. due to the density of light rays towards the Sun, astronomers located on earth, Mars and Venus would find sum of angles of Δ ever $> 180°$ ie gravitational field space around the Sun is +ve curved. On the other hand, for three observers A, B, C on the merry-go-round, the space would be −ve curved

According to Newton's theory, the Sun produced a gravitational field in the space round it that makes the planets move along curved trajectories. But Einstein's general interpretation of gravity is that it is due to the curvature of space-time [That the path of a planet in space time is a geodesic is shown schematically as explained by using space in two dimensions and taking time vertically. The curved line comes out as a spiral. Beautiful illustration on p. 104] . Einstein's theory of gravity leads to results different from those of Newton's theory even as regards motion of planets. For example, the former explains the difference between the calculated and observed rates of precession of the major axis of Mercury's orbit, which was a long-standing problem mystery of classical mechanics.

(5) Gravity waves — Analogy between Newton's law of gravitation and law of electrostatic interaction between charges, and Einstein's theory of the gravitational field & Maxwell's theory of the electromagnetic field suggests that an oscillating mass should give rise to gravitational waves just like an oscillating electric charge produces electromagnetic waves. In Einstein's famous article of 1918, some solns of the eqns of general relativity were obtained as representing gravitational waves travelling with the velocity of light. If they exist, gravitational waves must carry energy, but their intensity or the amount of energy they transport is extremely weak. For eg. the earth in its orbit round the Sun would emit 0.001 watt, which would result in its falling a millionth of a centimeter towards the Sun in a billion years! Eddington argued that these waves are fictitious and could be pure mathematical fictions that can be eliminated by a suitable choice of space-time coordinates. More thorough analysis indicates, however, that this is not the case and that gravitational waves, weak though they may be, are real.

Two years ago Dirac succeeded in quantising the gravitational field equations and showed that the energy of gravity quanta or "gravitons" is given by $E = h\nu$ as in the case of light quanta or photons. The spin of the graviton is, however, twice the spin of the photon.

? [Where is this? To find out the corresponding references]
Is this Dirac's paper of 1958 on gravitational equations in Hamiltonian form?]

Because of their weakness gravitational waves are of no importance in celestial mechanics. But might not gravitons play some role in the physics of elementary particles? Electromagnetic interactions involve emission & absorption of photons; similarly gravitational interactions are related similarly to gravitons — Three types of interactions :- (1) Strong interactions including e.m forces (2) weak interactions like β-decay of a nucleus,

in which an electron & neutrino are emitted & (3) gravitational interactions which are vastly weaker than (2).

The strength of an interaction is related to the probability or rate of the emission or absorption of its quantum. For eg. a nucleus takes 10^{-12} sec. to emit a photon; in comparison the beta decay of a neutron takes 12 minutes ie about 10^{14} times longer. It can be calculated (how?) that the time necessary for the emission of a graviton by a nucleus is 10^{60} sec. or 10^{53} years! This is slower than the weak interaction by a factor of 10^{58}.

Comparison with neutrinos [Ref. to Philip Morrison, Sc. Amr, Jan 56 on neutrinos] - Bohr's enquiry of 1933 of what is the diff. between neutrinos & gravitons. Leaving aside weak interactions, what about processes involving only neutrinos, say the emission of a neutrino-antineutrino pair by an excited nucleus? No one has detected such events, but they may occur on the same time-scale as the gravitational interaction. A pair of neutrinos would form with a spin of two, the value of the spin of the graviton calculated by Dirac (where?). All this is of course the sheerest speculation, but a connection between neutrinos and gravity is an exciting theoretical possibility.

(6) Gravity and Electromagnetism - Faraday failed to find any relation between gravity and electricity - Einstein, who was convinced that the electromagnetic field must also have some purely geometrical interpretation, tried for it tried to build up a "unified field theory" which was hard going & not elegant like gen. rel. Today fewer & fewer physicists are working at such theories, and most are convinced that the effort to geometrise the gravitational field is fruitless. It seems, at least to the author, that the true relation between gravity & e-magnetism is to be found in the theory of elementary particles - an understanding of why there exist particles with just certain inertial masses and not others - and of the relation between the masses and the electric & magnetic properties of the particles.

Compare the actual strength of the electrostatic and gravitational forces between a pair of middle-weight particles say pi mesons (pions). Computation finds the ratio of these strengths = e^2/m^2G (G = universal gravitational const) for any two particles and for two pi mesons the ratio is equal to 10^{40}. Any theory that claims to describe the relation between electricity & gravity must explain this ratio, which is a pure number. Such numbers like 2π, $5/3$ etc often turn up in theoretical formulae for they are usually small, unlike 10^{40}.

Dirac's proposal nearly 20 years ago that 10^{40} the figure 10^{80} is in fact not a constant, but a variable that changes with time and is connected with the age of the Universe

According to the evolutionary cosmology the Universe is 5×10^9 years or 10^{17} sec. The second and year are arbitrary units, and we should preferably choose an elementary time interval that can be derived from the basic properties of matter and light. A reasonable one is the length of time required by light to travel a distance equal to the radius of an elementary particle $= 3 \times 10^{-13}$ cm $/ 3 \times 10^{10}$ cm per second $= 10^{-23}$ sec. To express the age of the Universe in this elementary time unit, we have $10^{17}/10^{-23} = 10^{40}$ (pure number). Thus according to Dirac the large ratio of electric to gravitational forces is characteristic of the age of the Universe. Also Dirac assumed that e does not change with time, but G decreases & this decrease may be connected with the expansion of the Universe. When this idea was put forward, the solar system was thought to be about three billion years old. Edward Teller of California University has pointed out (where?)* an objection to this viz. that the earth would have been 50° hotter than boiling water during the Cambrian era, when well developed marine life existed. Assuming that the solar system is three billion or more years old, in which case the Cambrian oceans though hot, would not have been vaporised. So Teller's objection loses its force, provided that afterwards Cambrian plants and animals could live in very hot water. [* Phys. Rev. 73, 801, (1948)]

(7) Anti-gravity. — Parallel similarity between Newton's law of gravitation and the laws that govern electric charges and magnetic poles. If one can shield electric and magnetic forces, why not gravity — The mechanism of electric & magnetic shielding; consider a metal placed in an electric field, the free electrons move to one side of the material giving it a $-^{ve}$ charge and leaving the opposite side $+^{ve}$. This polarisation produced a new electric field which is directed opposite to the original field & thus the two can cancel each other — Similar explanation of magnetic polarisation due to the shielding effect arising from polarisation of atomic particles.

Gravitational polarisation on the above basis of shielding appears untenable since particles with $-^{ve}$ gravitational mass are unknown in Nature.

There is however another Kind of matter — antimatter — that in many ways is the reverse of ordinary matter including its electric and magnetic properties. Perhaps anti-particles also have negative mass. It appears theoretically possible to decide this by watching a beam of anti-neutrons (?) emerging from an accelerator after the gravitation field of the earth and observe whether the beam bends up or down, but an actual experiment is impossible of performance since in a kilometer of travel of the beam with velocity of light, the bending, whether up or down, would only be about 10^{-12} cm, the diameter of an atomic nucleus. Thus from an experimental point of view the sign of the gravitational mass of anti-particles remains painfully open.

It is also open from the theoretical point of view, since we do not have a theory that

relates gravitational and electromagnetic interactions. If a future experiment should demonstrate that anti-particles do have a negative gravitational mass, it will deliver a mortal blow to the entire relativistic theory of gravity by disproving the principle of equivalence. An anti-apple might fall up in a true gravitational field, but it could hardly do so in Einstein's accelerated space-ship. If it did, an outside observer would see it moving at the twice the acceleration of the ship with no force at all acting on it! The discovery of anti-gravity would therefore force upon us a choice between Newton's law of inertia, and Einsteins equivalence principles. The author earnestly hopes that this will not come to pass.

4/7/68 Article by G. Gamow – Phys. Rev. Letters (25|9|67), p. 759 on "Electricity, gravity and Cosmology" where ideas in (6) above are discussed with an alternative approach.

[les temps (τ) in French: time necessary for light to cover a distance equal to the "classical" radius of an electron and = 10^{-23} Sec. This is the unit of time used in (6) above. [con = 10^9 y = 3×10^{17} Sec

In this article in Phys. Rev. Letters, Gamow argues that the alternative approach is to investigate effects consequences of e^2 and not $\gamma(G)$ varying changing with time. (in $e^2/\gamma m_p^2 \simeq 1.24 \times 10^{36}$)

I may ask "why not m_p change with time?" — yes Gamow proposes m_e not (protons) see 26|7|68.

III Articles on Quantum theory of gravity I, II and III by Bryce S. DeWitt (Phys. Rev. Letters 31|7|67) in the Abstracts of Articles (A1-A2) – These articles to be read after searching corresponding Phys. Rev. Nos.

Article on Lagrange formulation for systems with higher spin by Shaw-jin Chang – Harvard Univ. Ref to article which deals with $S = 3/2$ and 2, and if there be no reference to my work, send him reprints.

Article on "Gauge problems in quantum field theory" by F. Strocchi, Palmer Phy. Lab, Princeton (Abstract A6, Phys. Rev. Letters 31|7|6967) – Report deals with quantisation of the e-m fields.

The reps of the Poincaré group (?) for massless spin one particles in quantum field theory are shown to be essentially different from the corresponding ones of the classical case.

[Any relation to Ramani's query ?]

Article on "Testing general relativity with laser ranging to the moon" by Ralph Bayer Lain, in Phys. Rev. Letters A2) - Also three more articles on same page of A2 on special and general relativity. one disproves Fock's method of reinterpreting the chaotic metric in Einstein's General Relativity & another by showing that this leads to disagreement with gravitational red-shift expts.

On A7 - Article on "Operator formalism for daughter trajectories in the Bethe-Salpeter eqn" by V. Chung & J. Wright - The word "daughter" is rather interesting.

On A10 - Article on Regge daughter trajectories in the B-S eqn by same authors.

P.R.L. of 25/9/67 - In A1, 2 articles on general relativity, in A2, one article

(Read up to Phys. Rev. Letters 2/10/67).

26/7/68

P.R.L (2/10/67) - p. 790 - Reference is made to "spinless fermions" - what does this mean?

" (9/10/67) - p. 880 - " to book "Quantum Electrodynamics" by R.P. Feynman, 1962.

" (16/10/67) - p. 913 - Article by Gamow on "Variability of elementary charge and quasistellar objects - Again as in P.R.L (25/9/67) discusses variability of $e^2 \propto t$, while γ remains const - Testing of this possibility by observing the value of the fine structure constant $\alpha = \frac{2\pi e^2}{hc}$ in distant galaxies already done, but no satisfactory verification - Suggests testing with QSO's (Quasistellar objects) - Reformulation of the problem of universal constants:- Assuming (as) "natural" system of units the velocity of light c, h and the elementary length λ ($\simeq 10^{-13}$ cm) which presumably do not change with time, leads to the following "true secondary constants":-

(1) The mass of the heavy particles

$$M \simeq h/\lambda c \simeq 10^{-24} \text{ gm}.$$

(2) The range of nuclear forces (strong interactions)

$$r_0 \simeq \lambda \simeq 10^{-13} \text{ cm}.$$

(3) The strength (depth of potential well) of nuclear forces

$$U_0 = hc/\lambda \simeq 10^{-4} \text{ erg} \simeq 100 \text{ MeV}.$$

(4) Fermi's constant of the weak interaction

$$g \simeq ch\lambda^2 \simeq 5 \times 10^{-49} \text{ erg cm}^3.$$

For purely electromagnetic quantities, which may be time-dependent, we have the elementary charge $e \sim \sqrt{t}$ (possibly), and $m_e \sim t$ (possibly), these the latter horrible assumption being made to keep

The "classical" radius of the electron e^2/mc^2 equal to the constant elementary length λ.

p. 922 – Anti-neutron production process $\gamma + p \to \bar{n} + d$ is described (?)

P.R.L (23/10/67) A2 – Article by Arthur Komar on "Gravitational supercanon as a generator of canonical transformations" .. Talks of the quantisation program for the general theory of relativity.

A6 – Article on "current algebra etc etc (?) & talks of $SU(2) \otimes SU(2)$ & so on.

" – interesting new article on "Algebraic classification of Regge poles" where it is proved that (a) massless "particles" transforming according to to infinite-spin reps of the 2-dimnl Euclidean group are necessarily "elementary", (b) the classification algebra of Regge poles, at vanishing invariant mass, is necessarily the Lie Algebra of the homogeneous L.T, $SL(2, \mathbb{C})$

A8 – Article "Conspiracy and Evasion: A property of Regge poles" by Elliot Leader, Cavendish Lab, Cambridge, England – Talks of weakness in our present methods of applying Regge-pole theory to relativistic processes.

A.9 – "A Connection between $SU(3)$ and $\otimes O(4)$" by D. G. Ravenhall et all – Talk of L.H.O & H-atom generalisation & implication for elem-particle symmetries – ~~Proton matter~~ Find the correct mathematical relation between these groups

A.10 "New formalism for the quantisation of a spin-$3/2$ field" by Herman Munczek, Dept of Physics, ~~North~~ Western Univ, Evanston, Illinois – Sp-no reference to my work sent him a reprint.

A.11 – A paper about Lie Algebras

A.12 – "Formal reconstruction of theory from experiment" by M. B. Halpern

P.R.L (30/10/67) p. 1049 – Article on "Exptl. comparison of the gravitational force on freely falling electrons and metallic electrons" by Witteborn & Fairbank (Stanford Univ, California)

Measurements of the gravitational force have been made only on bulk matter, neutral particles of ordinary matter, and photons. Morrison and Gold [in Essays on Gravity (Gravity Research Foundation, New Boston, New Hampshire, 1957), p. 45] have suggested that antimatter may be repelled by ordinary matter. While indirect evidence from virtual antimatter in nuclei, and short-lived antiparticles suggests that anti-matter has normal gravitational properties, $\simeq 0$

direct measurements have been made. A repulsive force would be particularly interesting, not only because it would violate the equivalence principle of General relativity, but also because it would provide a mechanism for the large scale separation of matter and antimatter in the Universe. In this article, in order to test the gravitational properties of antimatter, it was decided to compare the gravitational acceleration of positrons & electrons in the earth's field. This article which is the first half leads to the conclusion that "the force of gravity on electrons inside a metal is the same as that on electrons in a vacuum".

P.R.L (6/11/67) - p. 1095 - "Method for solving the Korteweg - de Vries equation" (KdV) viz

$$u_t - 6uu_x + u_{xxx} = 0$$

which appears in connection with shallow water waves, hydromagnetic waves in a cold plasma, ion-acoustic waves & ~~acoustic~~ acoustic waves in an anharmonic crystal — Purely mathematical & highly interesting general solns - Soln numerical solutions give "solitons" & "kinks" which are obtained here fairly.

p. 1154 - "Photons, mesons and form factors" by J. Schwinger.

P.R.L (13/11/67). p. 1171 - Article about self-focusing and self-trapping of intense light beams which have recently became one of the most important and important subjects in non-linear optics (?).

p. 1193 - Article talking about asymptotic $SU(3) \otimes SU(3)$.

p. A11 - "order of levels of symmetric Hamiltonians".

P.R.L (20/11/67). p. 1256 "Calculation of daughter trajectories"

P.R.L (27/11/67). p. 1291 - "Time variation of charge of the proton" by F. J. Dyson - Gamow proposes $e^2 \sim t$ --(1). Dirac originally proposed that the two dimensionless numbers e^2/Gm^2, $tmc^2/\hbar$ ...(2) which are presently both of order 10^{38} should increase proportionally as the universe evolves. Dirac supposed that e^2/Gm^2 would increase through a decrease of gravitational forces according to $G \sim t^{-1}$ -- (3) Many geophysical and astrophysical considerations argue against (3) & Gamow suggests (1) as a substitute. Teller suggested that e might decrease with t according to

$$(\hbar c/e^2) \sim \log (tmc^2/\hbar) \quad -- (4)$$

so that "137" would be roughly equal to the logarithm of the big numbers (2). I present here a simple argument from nuclear systematics which excludes both possibilities (1) and (4).

p. 1291 - "Constancy of e" by Asher Peres - Rules out the above possibility (1) of Gamow on the basis of geological evidence.

p. 1294 – "Does the fine structure constant vary with time?" by Bahcall and Schmidt – Proves by astrophysical evidence inferred from the spectra of radio galaxies with red shifts about 0.2 that α is constant, that Gamow's assumption of $e^2 \sim t$ is not tenable.

A1 – "Hamiltonians in relativistic classical particle mechanics" by T.F. Jordan

– "Sets of quantum state by measurements" by W. Gale et al – A question raised by Pauli whether or not the probability densities in space and momentum determine the wave function is answered negatively.

– "Momentum flow by gravitational waves" by F.I. Cooperstock.

A2 – "Vierbein field theory of gravitation" by F.A. Kaempffer

A7 – "Analyticity properties applied to obtain numerical solutions of the Bethe-Salpeter eqn"

A14 – "Nondynamical tests of the CPT theorem and related symmetries"

P.R.L (4|12|67).

PRL (11|12|67) – p. 1385 – "Finite translations in solid-state physics" by J. Zak. – Shows that finite translations in direct and reciprocal space can be chosen to form a complete set of commuting operators in quantum mechanics (Ref. Dirac's book). The eigenfunctions of these operators are found and are proven to form a complete orthonormal set of functions.

29|7|68:

(1) Phys. Rev. (25|11|67) p. 1353 – Symmetrischin postulate of quantum mechanics

(2) " p. 1361 – Method for finding the general-relativistic effective potential by Jong. K. Jaen [General-gravitat relativistic gravitational effects are is quite negligible compared with the Coulomb-electrostatic and Yukawa-nuclear potentials at intermediate distances, but quite dominant in the vicinity near the potential source. It is possible that we have overlooked this important gen-rel. effect in describing the structure of matter in the microscopic world in this reference to a view point from which one explains the nuclear force as a gravitational force (Barnothy J.M & M.F, Amer. J. Bull. Amer. Phys. Soc 12, 420 (1967))]

(2) 2 other articles on gen-rel in same issue

(3) Phys. Rev. (25|12|67), p. 1595 – Gravitational interenergy as a Generator of Canonical transformations by A. Komar

(4) " p. 1794 – New formalism for the quantisation of a spin 3/2-field by H. Munczek (no ref. to my work)

(5) Dublin Inst. of advanced studies — Aspects of the current algebra approach by C. Ryan
Series A, Vo.17 — Based on a seminar — In the Math. Shelf-upstairs in Library.

12/8/68

(1) Sc. Amer. July '59 — The Exclusion Principle by Gamow — Beautiful photograph of Pauli — Ref. to his receiving Nobel Prize in 1945 — Died in December, 1958 — Pauli Principle also leading to explanation of both electrical & thermal conductivity in metals — in white dwarfs, which are stripped to bare nucleii & electrons are torn from their customary orbits forming a free electron gas. This gas is quantised and the electrons are distributed among a gigantic number of quantum states according to Pauli's principle. Since the electrons possess much higher energies than the average thermal energy even inside these stars, the gas exerts a tremendous pressure which prevents the stars from collapsing — Dirac's explanation of quantum levels representing the extraordinary states (-ve energy) of elementary particles using the Pauli Principle — Nuclear structure & the quantised nucleus; consecutive shells inside the nucleus can accommodate 2, 8, 14, 20, 28, 50, 82 and 126 particles of either kind (± spin); these magic numbers of the nucleus were first found by observation & then explained by the Pauli Principle; use of the same to find antiprotons and anti-neutrons — Wave mechanics; full comprehension of Pauli's principle requires that it should ~~the theory~~ be understood not merely by above applications, but by ~~both~~ in terms of modern quantum theory; Pauli's first explanation of the Principle in terms of the "mathematical symmetry" of probability functions viz. the probability function describing the motion of an electron within an atom is antisymmetrical w.r.t any electron pair, as originally stated by Pauli in 1925 — Non-Pauli Particles; Non-Pauli particles (for eg. two helium nucleii, alpha particles) & Pauli particle (for eg. nitrogen atoms containing odd no. of particles) — Bosons and Fermions, the latter being Pauli particles; even in modern theory of elementary particles like addition of hyperons and mesons, the only concept that stands like the rocky Gibraltar is the Pauli Principle.

(2) Sc. Amer - Sept. '59 - Whole issue devoted to ionising radiation i.e. e.m. waves or material particles having sufficient energy for ionising atoms or molecules (removing electrons from them and completely altering their chemical behaviour); nuclear ~~weapons~~ weapons & increasing ionising radiation, hazards; moral issues also raised

1st article "What is ionising radiation" by R.L. Platzman.

16/8/68

(1) Phys. Review - 25/2/68 (a) — 2 papers by Isaacson on Gen. Relativity - Ref. to P.C. Vaidya's work, (b) Paper about "quarks" ie fractionally charged particles $\pm\frac{1}{3}e$, $\pm\frac{2}{3}e$, based on theoretical concepts of SU(3) symmetry, experimentally confirmed (Joint author B.V. Sreekantan, T.I.F.R); (c) Gauge problem in quantum field theory II - difficulties of combining Einstein eqns and Weinberg theory by F. Strocchi, p. 1302 - Reference to S.N. Gupta's article in Rev. Mod. Phys., 29, 334 (1957) & his book with Pergamon Press 1962) - Reference to group

the method work in gen. relativity like Poincaré group, ∞-non-abelian gauge groups - Result is that the repns of the Poincaré group for massless spin-2 particles in quantum field theory is essentially different from the corresp. classical case (d) Relativistic classical particle mechanics by T.F. Jordan - Ref to Poincaré group & Articles by Dirac in Rev. Mod. Phys 21, 392 (1949) & also reference to Sudarshan's Brandeis Summer Institute Lectures on Theoretical Physics published by W.A. Benjamin & Co, N.Y, 1962 - Reference to Wigner's book on Group Theory & Pontrjagin's Topological groups. (e) Phenomenological Lagrangian for field (Current or Current) Algebra, Hard Pions & Radiative Corrections by Lee & Nieh p.1507 - Talks on Current Algebra, PCAC (Partially Conserved axial vector current), Chiral SU(2) ⊗ SU(3) symmetry, SU(2) ⊗ SU(2) algebra etc. (f) 2 Papers one by R. Ramachandran & another by K.V. Vasavada & Prem Nath, pp. 1528 & 1532. (g) Non-linear realisations of Chiral symmetry by Steven Weinberg, p.1568 - Current algebra is useful because it allows physical predictions to be made from chiral symmetry. (g) Conspiracy and Evasion: Property of Regge poles by Elliot Leader, p.1599. (huge article of 21 pp.) (h) Subconvergence relations and the quark model by Donald E. Neville, p.1665 - Static SU(6) has been an unusually fruitful theory in that the attempts to make it relativistic have produced not one, but at least two theories. These two are the algebra of currents, and the quark model (References to both these); refers to difficulties of the quark model for eg. quark-anti-quark pairs etc. (i) Article by S.M. Vaidya (T.D.F.R), p.1676 - A physically interesting representation for baryon isobars for the group of strong-coupling theory $[SU(2) \otimes SU(3)] \times T_{24}$ was obtained by Goebel and others as an induced representation. It is shown that the Goebel repn cannot be obtained by a straightforward contraction on SU(6), but that it can be obtained by contraction on $SU(2) \otimes SU(6)$. (j) Paper by Sivakaran & Pandit, p.1782 (T.D.F.R) and another by Sivakaran, Gupta & Rajasekharan, p.1792. (k) An article on an experimental study of the Kapitza-Dirac suggestion that a standing light wave can serve as a diffraction grating for a beam of electrons. Experimental possibility nowadays using lasers. Results inconclusive, p.1494. — (l) - number of articles on Regge Poles.

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(2) Rev. Mod. Phys - April, 1968 - (a) Magnetic groups & their co-representations by Bradley & Davies p.359, (b) Phase & angle variables in quantum mechanics by Carruthers & M.M. Nieto, p.411

In (a) reference is made to a book by Bhagavantam on "Crystal symmetry and Physical properties" (Academic Press, Inc, New York, 1966) - Also references to P. Rudra & M.V. Murthy (J. Math. Phys, 65 & 66)

6/8/68

(1) Rev. Mod. Phys - Jan 1967. - (a) Article by P.P. Kane & G. Basavaraja (T.D.F.R) on non-linear quantum electrodynamic effects. (b) S-matrix Theory of electromagnetic interactions by Chou & Yennie

(2) Ibid - April, 67 - (a) Group Algebra, Convolution Algebra & applications to quantum mechanics by Per-Olov Löwdin - Quite elementary - must be read carefully!

(3) Ibid - July 1967 - Whole issue devoted to Proc. of an International conference on N-N (nucleon-nucleon) interaction called "The Mount Mount Everest of Nuclear Physics" - Gregory Breit presented with a Gold Watch.

(4) Ibid - October 67 - (a) Paper on 3-body problem in nuclear matter by R. Rajaraman and H. A. Bethe

(b) 2 articles on Self-adjoint ladder operators — The concept of a ladder operator represents an approach which gives rise to one of the most elegant methods for obtaining the eigen-values of a self-adjoint (Hermitian operator) - Origin in works of Dirac & Weyl

[Ref: "From the Clifford algebra $\beta_i \beta_j + \beta_j \beta_i = 2\delta_{ij}$, we define $\sigma_{ij} = (-i/2)[\beta_i, \beta_j]$ — What is this Clifford algebra ?

(5) Ibid - Jan, 1968 ||- (a) Regge Poles and SU(3): Particle classification and high energy scattering by V. Barger - must be read.

(6) Scientific American - Jan 68. - (a) g-factor of the electron by H. R. Crane

g-factor = ratio of electron's magnetic moment to its spin A.M. - Its precise measurement has had far reaching consequences

(7) -- Do -- Feb. 1968 - (a) Arrival of nuclear power by Hagerton - Electric power obtained from nuclear fission has made a decisive market breakthrough and now accounts for nearly half of all the new power-generating capacity being ordered by U.S. utilities, (b) Holography - a new kind of photographic technique.

(8) - Do - March, 68 - (a) Anti-ballistic missile systems by Garwin & H. A. Bethe

(b) Human cells & aging by L. Hayflick.

(9) - Do - April, 68 - (a) Photon echoes - 2 light pulses are directed into a ruby and 3 pulses emerge

(10) - Do - May, 68 - ||(a) The Three Spectroscopies by V. F. Weisskopf - Interesting article relating to elementary particles - must be read. (b) Photographs of surface of moon (c) An article on dextrality.

28/8/68 Comm. Dublin Inst. for Advanced Studies — Series A. No. 18

✓|| - A treatise on Gravitational radiation by Julian McCrea (100 pp)

2/9/68 - Phys. Rev. Letters - (8/7/68) July 68 - New formulations of General Relativity by Eugene Gaudi, p. 606

✓|| (Of Oak Ridge National Lab, Oak Ridge, Tennessee)

✓✓ || States "Agreement with expt. is necessary but not sufficient for a truly physical understanding of a set of phenomena" - Also ref. to Born-Infeld theory in the

The essential point of the note is to establish the uniqueness of GR by postulating the derivability of the eqns of motion from the gravitational field eqns. GR is the only theory in which the field eqns imply the eqns of motion without introduction of new constants (in addition to Newton's gravitational const). The containment of the eqns of motion in the field eqns is, in my opinion, one of the most remarkable examples of the "unreasonable effectiveness of mathematics in physics" - etc. etc - to be read.

4/9/68 (1) Sc. Amer - Jan 1957 (Vol. 196, 1957, p. 84 - Pions by R.E. Marshak - These particles also known as pi-mesons are an important structural material. They appear to be the cement which holds protons and neutrons together in the nucleus. - Last para: The chief difficulty is the fact that we must deal with a swarm of pions. Our mathematical techniques cannot effectively handle more than one pion at a time. Beyond this, things become much too complicated. The problem appears to be a basic one, and it seems that only some drastically new idea will enable us to solve it. And so the pion, while providing a tantalising glimpse into the nuclear force, seems also to deepen our ignorance.

(2) Ibid - April, 57, p. 45 - Overthrow of parity by P. Morrison - Conservation of parity said that there is no absolute distinction between right and left in nature. Exptl. now show that such a distinction exists - Mirror invariance not holding in weak interactions (eg. decay of the mu meson - Does conservation of energy is found to hold in the weakest interaction observed so far, but does it still hold for the weakest interaction of all, those involving the weak force of gravity? Here one thinks of the hypothesis that matter may arise spontaneously from a space containing no energy, and the possibilities are exciting. Perhaps violation of parity may also explain the near-exclusion of antimatter in our world, thus forging a bridge between the microphysics of fundamental particles and the physics of great distances in cosmology.

(3) Ibid - July 57 - Elementary particles by Gellmann - Makes clear the notions of isotopic spin & strangeness.

(4) " - Jan 58 - Principle of uncertainty by Gamow - Nothing new.

(5) Phys. Rev. Letters - 24/6/68 - On p. A-2 consisting of articles to be published in the Phys. Rev. Letters mention is made of an article "Supplementary conditions in the quantized gravitational theory" by Suraj N. Gupta, Dept of Physics, Wayne State Univ, Detroit, Michigan.

Abstract: The role of the supplementary conditions in the quantum theory of gravitation is discussed in a simplified and unambiguous manner by means of a Lagrangian formalism involving the use of the ordered products. The consistency of the supplementary conditions for the general nonlinear gravitational field interacting with the matter field is established here.

What is all this about? Ref. to his article in Rev. Mod. Phys & his book of 1962. (vide 16/8/68)

Does this mean that now the problem has been solved for Quantum

(c) Rev. Mod. Phys – S. N. Gupta's article is nothing but confining to flat space & talking of gravitons, without reference to Dirac's work mentioned in Gamow's Sc. Amer. article on Gravity.

6/9/68 – Rereading more carefully article by Gellman on Elementary Particles in Sc. Amer. of July 57.

9/9/68 (5) Rev. Mod. Phys. Jan 68, p.129 Regge Poles & SU(3) – Complete Greek – Talks of ρ-meson ?

(2) " p.140 Study of leptonic decays 3-body decays of baryons by M. M. Nath Nicto,

p.140 – Talks of Isospin, current-current form of the weak interaction of hadrons (?) &

(?) (?)

isotopic spin BSM 19/9/68 ← hadron current – Ref. to book by Cabibbo Vol II – Particle symmetries (Gordon & Breach Science Publishers, Inc, N.Y. (1966) – Ref to Weinberg, Phys. Rev, 112, 1375, (1959) & his terminology of F_1, F_2, G_1 & G_2 were associated with "first class" currents (proper vectors with G parity +1, and axial vectors with G-parity −1), F_3 & G_3 being associated with "second class" currents (G parity opp-te that of 1st class currents) → ? Ref. to book by D. Jackson: Elementary Particle Physics & Field Theory – 1962 Brandeis Lectures edited by K. W. Ford (W. A. Benjamin Inc. N.Y. 1963, Vol I) –

or J. Cole has performed a Monte-Carlo integration for the process $\Sigma^- \to n + \pi^0 + e^- + \bar{\nu}$ with imaginary data taken from a Dalitz plot" (What is all this ?)

Ref to C. S. Wu's article in Rev. Mod. Phys. 36, 618 (1964)

& book by H. F. Schopper – Weak interactions & nuclear Beta decay (North-Holland Pub. Co, Amsterdam, 1966)

(3) pp.77–129 – Huge collection of data on particles and resonant states – Summary updating that of July, 1967 (Rev. Mod. Phys, 39, 1 (1967)) – Colossal amount of data completely meaningless to me

(4) pp.228–35 – Some controversy about "hidden variables" in quantum mech (non-rel) on a paper by Bohm & Bub that "hidden variables" can exist – Fair reply to criticisms by B & B on p.235

(4) Journal of Math. Phys. Feb. 1968 – ~~It seems to be the right type of journal for such work~~ What I am thinking of ✗ BSM 17/11/68

11/9/68 – Reread Dirac's article in Sc. Amer. 1963, specially his proposal of difference of μ and e being explained by assuming e to be a sphere & μ to be next orbit stable configuration –

J. Math. Phys – Feb. 68

13/9/68 (1) Universitas Vol. 9, 1967, No. 3, p. 287 – Article on Gandhi – Image & Symbol of India by Dr Zenta Maurina.

(2) Physics Today – Vol. 21, No. 2, Feb. 1968, p. 101 – We hear That – An interview with G. Gamow (Born 1904, still living) – quite interesting – finally gives up the idea of variation of the fundamental constants (suggested by Dirac) supported recently by Dyson.

June 1968 (3) Ibid, p. 31 – The early Universe by Edward R. Harrison – Very interesting article bringing in fundamental particles and cosmology [TeV range ≡ E = 10^{12} – multi Tera range; Mesons & baryons jointly referred to as hadrons (strongly interacting); lowest mass hadrons are the pions]; halls of the Hadron Era, fall of dynasty, decline of the pion, the lepton era, ghostly neutrinos, the radiation era, the stellar era, Eddington's number, origin of galaxies, baryon inhomogeneity, matter & antimatter & lastly four kinds of information the last one being the high energy particle physics & says cosmologist should wake up & take this challenge. [66. references!]

(p. 99) Book on "Current Algebras & their applications" by B. Renner. Pergamon, Oxford, 1968. $9.00

" "Theory of finite groups. Applications in Physics: Symmetry groups of Quantum mechanical systems by Laurens Jansen, Michael Boon – North Holland (Interscience NY) 1967 $19.00!

p. 103 – We hear That – Abdul L. D. Landau (N. L.) – in ~~1959~~ 1962 – Also interviews with him (Born 1904 & died when he was 60.) – considered himself a pupil of Bohr – jailed for a year in 1938 as a German spy & was released only after Peter Kapitsa protested personally to the Kremlin! – Awarded Stalin prize 3 times – Tribute by Evgeny that Landau was a great writer, teacher & educator and at the same time an outstanding Physicist, a rare combination! Landau's jocular classification of physicists by way of a whole scale: Einstein (called 1st class) is 0.5, Bohr, Schrödinger, Heisenberg, Dirac & Fermi & some others to 1.00 & for himself he put it as 2.5 & later said he had gone up to 2.00. – Landau's appreciation of young physicists for eg. of Feynman – After the road accident on 7/1/62 (the blow of stones), which killed his wife, Landau & wife had textbooks of new type for school children, to draw up new national curriculum. Asked whether he had spoken with Einstein on "profundities" yes, but little. It was difficult to speak

with him since I did not interest him. Nobody interested him. He was too preoccupied with himself.

16/9/68

Jan 1968 — p.31 – Group Theory in Particle Studies by San fu Juan – Very interesting – must be read – Dirac matrices formig algebra D(12) could be extended to my algebra for n pairs 3/2 and 2. – Talks of δ-meson

p.75 – A hundred years of entropy by Mahadev Dutta (of D.D.T.?)

p.103 – Further tests to decide between gravity theories.

p.127 – A festschrift for Hans A. Bethe by Eugen Merzbacher – A picture of Bethe, very ugly face (as N. of Latest one)

p.161 – We hear that (i) P. Venugopala Rao of Georgia Inst. of Technology & Raymond H. Kleiss of the Univ. of Syracuse were appointed Asst. Profs. at Emory Univ., Atlanta, Ga.

(ii) Niels Bohr medal to Isador I. Rabi N.L.

Feb, 1968. p.101 – A talk with George Gamow.

March, 1968. – Special issue re. undergraduate curriculum.

p.89 – Book on "Birth of Growth of Ψ" Erwin Schrödinger: An introduction to his works by (1887–1961)
W.T. Scott – Reviewed –
A fine portrait taken perhaps when he was old.

p.112 – In "We hear that" an article on Infeld by P.G. Bergmann – Infeld died on 15/1/68 in Warsaw (Poland) – Quite interesting – must be about my age.

April, 68 – p. 34 – Gravitational waves by J. Weber. – on p.39 is given a list of formulas for curvature in general relativity

p.52 – Spectroscopy, Quantum Chemistry and Molecular Physics by R.S. Mulliken (N.d. in 1966 for Chemistry)

p.119 – Again some particle physics – p.121 – Nishijima's theory of the two-pion decay of the long-lived K-meson is supposed to be of third order, accounting for its CP-odd character & its reduced rate.

May, 1968. A special issue on "Twenty years of Physics" – To be read, specially The first article on Particle Physics by A. Pais. (*)

Collection of Problems in Mechanics by D.V. Meshcherskii – Useful for exam. work.

19/9/68 References (*) in Article by San fu Juan, 1968

(1) V.F. Weisskopf – Physics Today 16, no.6, 26 (1963)

(2) L.M. Brown – ibid 19, no.2, 44 (1966)

(3) M. Gellmann – Phys. Rev. 125, 1067, (1962); Physics 1, 63 (1964)

Physics Today (*) (1) Probably (2) 4 forces and 2 neutrinos, (3) Hadron symmetries.
underlined in this article on p.25, Pais talks of a pair of conservation laws: one for μ number (=1 for μ^-, ν_μ; =−1 for μ^+, $\bar{\nu}_\mu$; 0 otherwise) and one for e number (=1 for e^-, ν_e; =−1 for e^+, $\bar{\nu}_e$; 0 otherwise), and

that the conservation of pn number + the number = lepton number was known for a somewhat longer time;

on some page under hadron symmetries: Gives the formula $Q = T_3 + N/2$, where N is the nucleon-number (T_3 being I_3 or Swartz ie component of isotopic spin in a preferred direction) and says this form of Q applies to both nucleon and π [from $Q = e(I_3 + b/2 + S/2)$ we have for nucleon $S=0$, $b=N$, in this notation & hence empir is correct for π^+, $S=0$, $N=0$ & $I_3 = 1$ giving correct result & sign for π^{-1} and π^0]. T & T_3 are conserved in strong interactions, while e-m violates T, conserves T_3. Is this a preferred 3-direction in some space? For the present we do not know better than to say that T "behaves as" an A.M, in an abstract sense, made mathematically precise by saying that we deal with the T components as generators of an invariance group SU(2) — In 1952, the first π-nucleon resonance (the 3-3 resonance) had been discovered. In this region the cross-section ratio for $(\pi^+ + p \to \pi^+ + p : (\pi^- + p \to \pi^- + p) : (\pi^- + p \to \pi^0 + n)$ is about 9 : 1 : 2. This would correspond to production of N^* particles, with appropriate charge, by way of $\pi + N \to N^*$ (N = nucleon provided that N^* has $T = 3/2$. The N^* decays rapidly into $N + \pi$ again. We consider this N^*-scattering resp. as much as a particle as N (just as we might speak of the excited state of a H-atom as a much of a particle in its ground state). Since the N^* resonances, many more higher nucleon resonances have been discovered & all their resonances represent but one facet of the richness of structure that has been uncovered these past 20 years. (life time of resonances 10^{-23} sec.)

(cf) A new spectroscopy — Associated production: for eg: $\pi + N \to \Lambda + K$ (strong interaction) and (weak decay) $\Lambda \to N + \pi$ were de-coupled — All hyperons eventually decay into nucleons & so nucleon conservation became baryon conservation believed to be absolute — Baryon spectroscopy can be characterized as follows: for each value of Y (hypercharge) many resonances exist having various T and other quantum numbers. They decay strongly, conserving Y_1 towards their "Y ground state", then weakly on to the baryon ground state, the nucleon. And it is very much the same story with the mesons. More of this new spectroscopy is yet to come

Conservation and violation rules — The latter may exist if conservation laws are not obeyed. for eg. in weak force $\Delta Y = \pm 1$ only & for the same force $\Delta T = 1/2$ — Search for higher symmetries. This was tempting to find more intimate connection between Y and T, with its SU(2) group. Eventually it was found that the embedding of Y and T in the group SU(3) leads to a very useful further ordering of the hadrons. To avoid strange degeneracies if SU(3) were exact, it is assumed the group is approximate, and along with it come specific violation laws, for eg. the way in which SU(3) multiplets split up. In this way, sets of isospin multiplets could be seen to be connected as [illegible] protons-n-b SU(3) multiplets. Discovery of Ω^- [illegible] predicted in order to fill an SU(3)

of the SU(3) multiplet was a beautiful confirmation of the usefulness of SU(3). — No connection known between hadron symmetries and space-time symmetries. Some type of "dynamical symmetries" have appeared in particle physics (analogous to the static Russel-Saunders coupling). Probably SU(6) is of this static variety. It has enabled us to recognise connections between SU(3) multiplets of different spin. One of its interesting predictions is the value $-3/2$ for the magnetic moment ratio of proton and neutron. — Dynamical symmetries may be quite useful. As in the "bootstrap approach", it is likely that symmetries like isospin may after all be dynamical in origin. These and other theories which go by names such as "collinear", "coplanar symmetry", and "non-compact group symmetry" may have their roles to play in future developments. — Hadron physics is not at all a study of resonances only. In the region of resonances now known to go beyond 2 GeV/c^2. But there is a transresonance region which is less known (i.e. more regular). In this region, new and important regularities have been found about energy- and momentum transfer dependence. Another major source of information on strong forces comes from the use of e-m and leptonic probes of hadrons, from which we have learnt much about form factors. Also the discovery of antinucleons in 1958's, and other antibaryons later on has added further information. — An issue under debate is whether current hadron physics is a chemistry in terms of a more fundamental triplet of particles, as yet unseen. The future will decide.

9/9/68.

(4) Hadrodynamics. — No analogue known to exist in hadrodynamics to the Maxwell-Lorentz eqns of electrodynamics, in which we have a small parameter, the fine structure constant α, in terms of which one can expand. No equivalent to α in hadrodynamics; so while can postulate field eqns for strong phenomena, we do not know any approximate solns. — Greatest dynamical progress has been made in electrodynamics during the last 20 years. New techniques of "renormalisation" has led to extraction of meaningful finite answers to higher orders of α than had been thought possible by most (hastily tackling the infamous infinities of field theory). High precision measurements such as the μ-meson $g-2$ expt (?) are of crucial importance for such questions. The $g-2$ was known to 3% in 1958 & is presently known to 1 part in 10^{-3}. — Hadrodynamics also has moved forward using (i) only the general axioms of field theory, (ii) trying to see how far one can go with quite general postulates like causality and unitarity, not necessarily presupposing the existence of a field theory. — The approach using (i) & (ii) is commonly known as dispersion theory; it started round about 1955 with the forward π-nucleon dispersion relations. Next, non-forward relations were conjectured & partly proven; double dispersion relations, with in energy & momentum transfer, were also conjectured. — Essential content of such relations are analyticity properties of amplitudes w.r.t (i) to some sufficient analyticity as (ii) assuming it & as key for the consequences. Thus the analytic property called "crossing symmetry" connects the amplitudes for $a+b \to c+d$ (s-channel), $a+\bar{c} \to \bar{b}+d$ (t-channel), $a+\bar{d} \to \bar{b}+c$ (u-channel), where $\bar{b}$ is the anti-b. Using Feldstein expansions & assuming analytic continuation feasible, much progress was made — one

important example is the Regge pole hypothesis, according to which families of resonances with the same internal quantum number but different spin are dynamically linked. By crossing the high energy behaviour in the s-channel comes out, in the simplest instance, as a power law, with the exponent being related to the occurrence of such resonances in the t and u - channels.

(5) C, P, T : weak dynamics — (i) discrete symmetries — C invariance, crudely speaking, means that physical laws should be unchanged if all signs of intrinsic multipole moments (electric charge, mag. moments, etc) are reversed — discovery of violation of P in weak interactions in 1956 from β-decay studies, immediately followed by discovery of a similar effect in π-μ decay. It was found these also violate C, but some degree of order was restored by the realisation of the invariance, to considerable accuracy, under the combined operation CP — It is still to be seen whether this development presages new views on space-time description — one invariance closest to pure space-time properties is CPT invariance, which we believe to be rigorously true. Delicate checks on this are not far off. If CPT invariance were to fail, then I would say all hell will break loose — Related to CP invariance (or so called 'γ_5 invariance') is the 2-component theory of the neutrino, which states that neutrinos couple with only one of their two possible helicity states. Also CP invariance yields much information about the properties of perhaps the most subtle system in particle physics (K^0, $\bar{K}^0$). This is a pair of particles with a mass difference that is $\sim 10^{-15}$ its mean mass, actually observed — In 1964 came the next blow. CP invariance also turned out effectively to be inexact, as was shown from a small violation in the neutral K system. This time no new kind of order has yet been reestablished. It is perhaps the biggest challenge today to find out, even phenomenologically, what goes on. This effect violates our sense of elegance and simplicity. Its interpretation may well be profound.

(ii) currents, currents everywhere — It has been found for most weak interactions (and may be true for all) that they are effectively four-vector currents of hadrons and leptons coupled to other such currents. The hadron currents have a vector part V_λ and an axial-vector part A_λ. This V-A theory settled a somewhat puzzling state dating from the 1930's — More learnt about V_λ & A_λ. Firstly, V_λ has been found to be related, essentially by a simple proportionality, to part of the electromagnetic current. As a result some weak effects can be expressed in terms of e-m parameters (weak magnetism). Secondly (skipping technicalities) we have learned that the divergence of A_λ is [illegible] the π-field; in many ways, we can give a fair prediction of the π-μ decay rate in terms of this known parameter. Thirdly, V_λ and A_λ are current operators in the quantum mechanical sense. Of late, interesting studies have been made of the commutation relations of these kind of quantities, a subject known as current algebra. Important, it has been possible to [illegible] Using this we can calculate the absolute rate of the Cabibbo-Feller and the Fermi constants of β-decay in terms of each other. Thus we can check consistency of existing information. More [illegible]

"soft pion" results have been obtained that relate reactions involving 2 π-mesons to reactions involving one π less.

(6) Epilog – Gone are the days of a Faraday and his Anderson, of a Rutherford and his Kay. This is the time of team effort. To benefit this to the best the second law of thermodynamics. Nor does it signify the end of the highly individual scientist – The harvest in these 20 years has been rich, new particles, a new spectroscopy, the new symmetry structure from force to force, hallowed notions of C, P, T; the beginnings of a new dynamics – As to what next – who can say? What we badly need is greater synthesis. Perhaps there will be paradoxes that will lead us to revise our very basic concepts. Perhaps we will realise that paradoxes are already with us. In the 20 years we have had the rich fun of living through the beginnings of a great new adventure. There is more to come.

Next article on The Nucleus by D. Allan Bromley is also interesting – Prologue – Nuclear Morphology – nuclear structure (shell models, collective models & inter-relationships of models, new collective models, optical models, isospin states) – Nuclear reactions – New isotope elements – High-energy studies – A microscopic laboratory (here mention Lee & Yang's theory and crucial experiments by C.S. Wu and her collaborators proving non-conservation of P. It was noted that the culprit in the weak interaction might well be the neutrino if it had definite helicity of −1 or in other words moved as a left-handed screw. This conjecture also later experimentally later and found correct.– A definite charge dependence of the parity non-violating part of the strong interaction – Tests for time-reversal invariance recently made) – Conclusions (Attack on nuclear many-body problem is well under way, but a long road yet remains for us in the 20 years to come) – In the Prologue talks of strong-interaction physics which deals with systems having $\hbar \leq 1$ (Compton wavelength?) (?) & further says that at distances of the order of a nuclear Compton λ, the strong interactions are extremely strong of the order of GeV (?) and operable only between nucleon pairs; their effect is repulsive, thus preventing the formation of elementary particles with $\hbar > 1$"!

23/9/68 Physics Today – March 1966, (Vol 19, No 3), p. 23 – Article by T. D. Lee on C P T – Very interesting & difficult.

Ibid – Feb, 66, p. 44 – Article by L. M. Brown on Quarks & ways to particle symmetry.

25/9/68 - Mainly looked at what looks to borrow & finally borrowed one

27/9/68 (1) Rev. Mod. Phys. April 67 - "Group algebra, convolution algebra and applications to quantum mechanics" by Per-Olov-Lowdin. p. 259

Treats of Group representations from quite an elementary level restricting himself to finite groups

(2) p. 463 - Physical axiomatics by Mario Bunge - Philosophical questions.

(3) Ibid, Jan 67, p. 143 - S-matrix theory of e-m interactions by x & y - ~~no~~ p. 152

talks of Møller scattering & Bhabha scattering.

(4) Physics Today - 1967 issue - Bound volume.

Jan 67, p. 55 - Gravitational theory & observation by R. H. Dicke - Nearly 60 references!

p. 60 - Refers to P. Jordan's "new-scalar theory" (Schwerkraft und Weltall - Bonn. Ges. Z. f. Phys. 157, 112 (1959)

as a generalisation of the 5-dimensional general theory of Klein & Kaluza, and also to Dirac's cosmology (Proc. Roy. Soc. A. 165, 199, 1938.

p. 152 - A ~~fine~~ gratuitous & blue (ultraviolet) poem:

Hubble, Hubble
Toil and trouble
Cosmos burns and cosmos bubble

Around about the redshifts go.
Can they be distant, yes or no?
Do we abide in a steady state?
Is matter made at even rate?
Divinely self, ex nihilo?
Round the C-field caldron go.

Within the Schwarzschild radius
Does central heating wait for us?
Or are we slowly burning out
While quasistellars wheel about?
From primeval do we fly,
And with a whimper shall we die?

So played the music of the spheres
Now is naught but discord to our years
(Or are there notes that we don't know
And themes that only time will show?)
Won't someone take our entropy
And find out what our fate must be?

Hubble, Hubble

— Det.

Feb, 67 - We hear that, p. 95 - Obituary

Subramanyan Chandrasekhar, professor of theoretical astrophysics at the Univ. of Chicago was awarded one of the eleven 1966 National Science Medals. The medal is in recognition of his

March, 67, We hear that, p. 135 – Murray Gell-mann has been named R. A. Millikan Prof. of Physics at the Calif. Inst. Tech. He is the first man to hold this chair created last fall. He joined the Cal. Tech. staff in 1955 as an associate professor. A year later at the age of 26 he was made a full professor (ie he is just 42 years old now). He is well known for his contributions to elementary particle physics; in particular he introduced the important ideas of strangeness, the eight-fold way and the quark model (A nice picture with glasses on).

April, 67 – We hear that, p. 110 – A biography of J. R. Oppenheimer (Born 22/4/1904 and died 18/2/67). by L. I. Schiff (Stanford Univ.)

May, 1967, p. 23 – Nuclear structure & modern research by V. F. Weisskopf

p. 27. Non-linear problems in physics by Werner Heisenberg (Received Nobel Prize for his work in 1932). Since 1941, he has been director of the Max Planck Inst. for physics & Astrophysics, Munich – Non linear problems play a central role in physics & even quantum theory which is linear may ultimately have to be replaced by a non-linear one.

June 67 – An Institute in North Bengal (p. 44), by E. M. Hafner – How can one learn to do things better next time? – Talks of language difficulty, miserable English spoken & so on & so on, all perfectly true. On p. 50, praises the Bhagavad Gita, finds a quotation for Karma Yoga viz "Endeavour always, but without attachment. That is how man reaches the Ultimate truth: by working without anxiety about the results?" Also mentions that J. R. Oppenheimer (recently dead in 67) was a deep student of Hindu literature, and on the day of Trinity recalled words from the same source" of the radiance of a thousand suns were to burst in the sky, that would be like the splendour of the Mighty one .... I am become Death, the destroyer of worlds" – The author's interest is in short-wave music

Read again when necessary

p. 103 – We hear that – G. N. Ramachandran, an Indian authority on protein structure has become professor of biophysics at the Univ. of Chicago. He will retain his position as prof. of physics at the Univ. of Madras

– Blackett N.L., President R. S. – recently elected Hon. member of the International Academy of Astronautics

p. 105 – J. C. Slater receives 1967 Langmuir Prize ($ 5000)

– Rosenbluth " Albert Einstein Award ($ 5000) – one of the Scientists who developed the H-bomb

July 1967, p. 31 – Article on the Mössbauer effect

p. 115 – C. S. Swarty has become editor of Physics Teacher

August 67 – P. Bergmann joins an award (p. 77)

Sept. 67, p. 54 – An article on Thomson's dilemma of Lord Kelvin – did the second law of thermodynamics require processes that did not exist?

on p. 55 is a beautiful portrait of W. Thomson (born 1824 – 1907)

October, 67, pp. 34-56 – Memorial to Oppenheimer – with wisdom, humour and affection, one man inspired a generation of physicists – Tributes by Robert Serber, V. F. Weisskopf, Abraham Pais and Glenn T. Seaborg

✓✓ p. 57 – Comment on Indian Institutes by people (S. Ramans S. N. Sen, R. D. Sharma & K. V. Rao) – Do we fail to help others help themselves?

Some say yes, but some say no. – This is a commentary on Harper's article on "An Institute in North Bengal" which was a failure, by people agreeing and disagreeing with him – Final reply by Harper confirming his opinion

Nov. 67, p. 23 – Relativistic Cosmology by W. Rindler

✓ Quasars, primordial radiation and the blobs can raise many new questions

p. 55 – Physics in France – Thanks to X, Y, Z, France suffers no brain drain

p. 125 – Julian Schwinger (N. L.) distinguished Prof. at State Univ. of N. Y.

p. 129 – Sheldon death of Sir John Cockcroft, N. L. & Cambridge physicist at age of 70 – Account of his career

p. 131 – A nice portrait of Cockcroft & Gamow at Cavendish Lab (Dec. 1933)

Dec. 67. ✓ p. 42, The visionary ideas of Bernhard Riemann

p. 105 – H. A. Bethe wins 1967 Nobel Prize in Physics – Account of his work

p. 107 – Walter S. Chamorasekhar named Perkins Prof. of Physics

p.109 - Franklin Prize awards to Gell-mann - A short account of his work.

- I.I. Rabi, S. A. Goldschmidt of France & W. Bennet Lewis of Canada were awarded the 1967 (of U.S.A) recipients of the Atoms for Peace Award

30/9/68 Looked at books on Relativity in Section of & was impressed by a recent book "Principles of Relativity Physics" by Anderson (acquired by I.I.Sc. Library in May 68) — To be borrowed later.

p.297 (8-10) The gravitation field in special relativity.

p.358 — Dirac eqn in a gravitational field.

p.437 - Dirac-particle eqns of motion in general relativity

4/10/68 — Looked at the Proc. of Th. Physics published by Gauthier Villars on general relativity — Also an article by Feynman in Acta. Phys. Polonica — Looked into 2 issues of Phys. Rev & P.R.S, but could not find the article by Dirac referred to by Gamow — Also found that S.I.A.M. Rev. is not in the library, but only the Journal.

7/10/68 · Heisenberg's article on non-linear problems in Physics — Physics Today - May 1967, p.23 — Further extracts: I have come across, on my way through Physics, a few non-linear problems so, I at least know some of the horrible difficulties and troubles which one meets in these problems — Einstein's remarks that Maxwell eqns in empty space represent formulations which are linear in form only because they are based on experiences with very weak electromagnetic fields." In general linear laws fulfil the superposition principle for their solutions but contain no assertions concerning the interaction of elementary bodies. The true laws cannot be linear, nor can they be derived from such" In 1953 Heisenberg published a paper on quantisation of non-linear eqns. This work has been followed by many papers on the quantum theory of such wave eqns & non-linear spinor theory —

Four characteristic features in many non-linear problems: (1) we can always use the symmetries to simplify the problems, (2) linearisation around special solns, (3) Solns have a kind of unpredictability in you never know what the solutions will finally do, and it may not be possible to extend the solns beyond a certain time. & finally (4) a way out is statistical methods in which we can be interested, not in special solns but in ensembles of solutions that we do not know completely — Discussion of

problem of viscous fluid motion brings out these four points — Is quantum theory is linear or a non-linear theory It is linear in its [illegible] but is linear in the sense that although the equation of motion are non-linear, this can be fulfilled by solving Schrödinger eqns which are definitely linear — Linearity in Q.M. has a very deep, almost philosophical reason & is not just connected with some approximation. The possibility of adding two solns to get a new soln is absolutely essential for the whole foundations of Quantum Theory. Linear character of quantum mechanics is not approximate in the sense that linearity of Maxwell's eqns is approximate — Three body problem of He-atom can be solved in quantum theory & going to limit $\hbar \to 0$, we should get soln of the classical 3-body problem which is non-linear; in general it looks as if we can solve linear eqns in Q.M & going to $\hbar \to 0$, solve non-linear classical problems, but this leads only to ensembles of solns to the classical problem; if we first let $\hbar \to 0$ & size of wave packet $\to 0$, and then go to larger times we do get a complete reply of the classical theory from quantum theory. If, however, we reverse the limiting processes, first go to ∞-times & then let $\hbar \to 0$, the situation is different because the wave packet has become infinitely big in every case. In spite of this connection between non-linear classical theory & quantum theory, the latter is definitely linear — Is this still true when we come to the theory of elementary particles — Mention of non-linear spinor theory — Cases of $\gamma_\mu \frac{\partial \psi}{\partial x_\mu} + k\psi = 0$ —(1) and $\gamma_\mu \frac{\partial \psi}{\partial x_\mu} + \psi(\bar{\psi}\psi) = 0$ —(2). eqn (1) leads to the linear Schrödinger eqn, but the non-linear eqn (2) can be replaced by a Schrödinger eqn which is neither a linear diff. or integral eqn, but a linear functional equation — Appearance of extremely complicated non-linear integral eqns, and the trouble is we do not know whether we really have to solve these equations (!) — The present conclusion is that we do not know whether ultimately the fundamental problem in the quantum theory of elementary particles will be a nonlinear problem or a linear one.

E. P. Wigner. Rev. Mod. Phys. 29 (1957), p. 255.

His [illegible] that Quantum mechanics has not yet been fully acquired to the postulates of the special theory of relativity is perhaps arbitrary. It does not alter the fact that the question of this consistency of the two theories can at least be formulated, that the question of special relativistic invariance of quantum mechanics by now has more meaning than at the time of its formulation

(R) This is not so with general relativity. The basic premise of this theory is that coordinates are only auxiliary quantities which can be arbitrary values for any event. If the coordinates can be given any values one wants & so also the momenta. If we want to find General relativity we [illegible] terms into quantum mechanics, one finds that

is to bring the statements of the general theory of relativity into such form that they conform with the basic principles of the general relativity theory which

Quantum limitations of the concepts of general relativity: The metric in the general theory of relativity is a metric in space-time, its elements are distances between space-time points, not between points in ordinary space. The events of the general theory of relativity are coincidences, this is, collisions between particles. The founder of the theory, when he created this concept, had evidently macroscopic bodies in mind. Coincidences, ie collisions between such bodies are immediately observable. This is not the case for elementary particles; a collision between these is extremely much more evanescent.— The essentially non-microscopic nature of the general relativistic concepts seems to us inescapable — The situation is still less satisfactory from a more fundamental point of view. It remains true that we consider, in ordinary quantum theory, position operators as observables without theorizing what the coordinates mean. The concepts of quantum field theories are even more weird (than those of gen. relativity) from the point of view of the basic observation that only coincidences are meaningful. If we analyse the way in which we "get away" with the use of an absolute space concept, we simply find that we do not. In our experiments we surround the microscopic objects with a very macroscopic framework and observe coincidences between the particles emanating from the microscopic system, and parts of the framework. This gives the collision matrix which is observable, and observable in terms of macroscopic coincidences. However, the so called observables of the microscopic system are not only not observed, they do not even appear to be meaningful. There is, therefore, a boundary in our experiments between the regions in which we use the quantum concepts without worrying about their meaning in face of the fundamental observation of the general theory of relativity, and the macroscopic region in which we use concepts which are meaningful also in the face of the basic observation of the general theory of relativity but which cannot be described by means of quantum theory. This appears most unsatisfactory from a strictly logical standpoint.

Ibid, p. 497 - Feynman quantization of gravity by C. W. Misner.

Section 2 - Four approaches have been applied to discover the content of the quantum theory of general relativity: (1) one considers an ideal Lorentz space, and treats the departures from it of the actual metric from this flat condition as small — ie one linearizes the gravitational field equations, applies standard methods of field quantization [Ref.

(i) L. Rosenfeld, Ann. Physik 5, 113 (1930), (ii) W. Pauli and M. Fierz - Helv. Phy. Acta. 12, 297 (1939)

and then attempts to modify this basically linear theory by reinstating the non-linear terms

[Ref (i) F. Belinfante, Revista. Mex. Fis. 4, 192 (1955, (ii) B.E. Lautrup, Nuovo Cimento (10), 4, 1445 (1956]. All other approaches consider the field eqns of general relativity in their fully covariant but non-linear form. (2) The canonical formalism endeavours to investigate the non-commutative algebra of the factors of the theory in general and the Hamiltonian operator in particular, with a view to finding ultimately in this way eigenvalues and transition probabilities [Ref (i) P. Bergmann, Rev. Mod. Phys. 29, 352 (1957), (ii) Belinfante, Caplan and Kennedy, Rev. Mod. Phys. 29, 518 (1957), (iii) B.S. DeWitt, ibid p. 377. one sets up an appropriate algebraic structure on the field variables by defining Poisson or Dirac brackets [Ref. Dirac, Can. J. Math. 2, 129 (1950); 3, 1 (1951]. Here "appropriate" is an abbreviation of two conditions: (a) the algebraic formulation of the classical theory is equivalent to the Lagrangian formulation, (b) the algebraic structure admits a rep^{n} by linear operators in Hilbert space where the brackets are represented by commutators. The problem thus defined has next to be solved by finding a particular operator representation.

The other two approaches ~~(3) and (4)~~ deal with propagators $(t_2\sigma_2 | t_1\sigma_1)$ that are natural generalisations of the propagator

$$\langle x_2 t_2 | x_1 t_1 \rangle = \left[m/2\pi i (t_2 - t_1) \right]^{1/2} \times \exp . i m (x_2 - x_1)^2 / 2 (t_2 - t_1) \quad -- (1)$$

of the simple one particle problem. The hypersurfaces σ_1 and σ_2 take the place of the time variable t_2 and t_1; and the field configurations on these surfaces f_2 & f_1, take the place of the position coordinates, x_2 & x_1. (3) The action principle of Schwinger [Phys. Rev. 82, 914 (1951)] deals with the infinitesimal changes in this propagator

$$\delta \langle t_2\sigma_2 | f_1\sigma_1 \rangle = (+ i/\hbar) \langle t_2\sigma_2 | \delta I | f_1\sigma_1 \rangle \quad -- (2)$$

which come about through changes in the action I, either by way of alteration in the position of the two surfaces, in the configuration of the fields on the surfaces or otherwise. In this it is like the canonical formalism using Hamiltonian & momenta, Schwinger's method deals with the action, and expresses itself in a manifestly covariant form. This method has not been applied independently to general relativity, but is used with in conjunction with Feynman's method (4) the Feynman method focuses, not on a differential equation to be solved for the propagator, but on a formula for the solution

$$(t_2\sigma_2 | t_1\sigma_1) = N^{-1} \sum \exp (i I_\mu) \quad -- \quad -(3)$$

with this entry goes a formula for a matrix element which Dyson [Adv. Quant. Mech. - Mimeographed lecture notes, Cornell, Ithaca] writes in the form

$$(\sigma_2 | \Theta | \sigma_1) = N^{-1} \sum_H \Theta_H \exp(i I_H) \quad - - \quad (4)$$

where H denotes a field history i.e. a definite specification of the field throughout the region between the hypersurfaces σ_2 and σ_1. The quantity Θ is a function of the field and Θ_H its value for the particular field history H — like the Schrödinger method, the Heisenberg method deals with manifestly covariant quantities — the inappropriateness of the linearized treatment! — In this version, it is easy to make calculations by standard perturbation methods. However, the constant length L^* [$= (\hbar G/c^3)^{1/2}$ $= 1.62 \times 10^{-33}$ cm] is so small (cf. the distances relevant in any familiar experimental context (10/68) that such computations are not of much interest. If gravity is to occupy a significant place in modern physics, it can do so only by being qualitatively different from other fields. As soon as we assume that gravity behaves quantitatively like other fields, we find that it is quantitatively insignificant — In classical theory the linear approximation makes sense — not so in quantum theory. We can form a state which makes the initial surface σ nearly flat, even on a sub-microscopic level. However, uncertainty relations will be expected to prevent our simultaneously restricting the time derivative of the metric at σ to a small value. Consequently, large curvatures over small regions are to be expected. Similarly, even if there were a "vacuum state" for the gravitational field, there is no reason to expect that it would be a state where the metric is nearly flat over small regions — a point emphasized by Prof. Wheeler. In other words, a linearized version of general relativity seems precisely adapted to throwing out just those features which are physically new and interesting. We therefore pass it by.

Ibid, p. 353. Summary of Chapel Hill Conference 1957 on Relativity. by P. G. Bergmann.

Wheeler's geons having only mass as the value, but no rigorous notion of field eqns with properties similar to geons (which obtained by a kind of self consistent approach; a collection of particles or waves which is described in terms of statistical assemblies, with random phasing, serving as the source of a gravitational field) — Existence of gravitational waves; no rigorous notion of field eqns representing those so far obtained — Prospect of unified field theories; Wheeler considers that it may well be that all the physical fields known to us at present will turn out to be manifestations of but one field, the original gravitational field of Einstein's theory of 1916. Personally, I am convinced of the desirability of some unified field theory; but I have the impression that right now we are obtaining so much information about new fields and new types of particles that it may be

advisable to wait a few years before making a new determined effort at unification.

Quantum theory of gravitation. Basically we feel the need for such a theory since we cannot reconcile the idea of classical & quantum fields existing side by side. More particularly, it appears unlikely that a mass point that gives rise to a classical Schwarzschild field is itself subject to Heisenberg's uncertainty relations. This, I believe, is the principal argument in favour of quantisation. On the positive side, we may perhaps hope with Pauli that quantising the metric tensor may eradicate the infinities of the other field propagators along the light cone and that it may make a contribution to the theory of elementary particles — Approaches can be divided into those taking their departure from some canonical or Lagrangian formulation, and those that attempt a path integral quantisation — For the former the construction of true observables [~~what observation~~ a true observable is a physical variable is independent of the choice of the co-ordinate system, gauge frame and the like] Work by Gréhéniau, A. Komar, E. Newman & P. Bergmann and others going in that hope that leading towards the true observables in the theory of gravitation — For the latter there are two schools viz (i) those believing that true observables are a prerequisite for this method & (ii) those like Feynman who propose to go ahead & calculate formal expressions in terms of the original $g_{\mu\nu}$ in the hope that these expressions will automatically be useful propagators. Such calculations involve enormous labour & lead to results of such complexity that will not be enlightening — Wheeler's wormhole picture is here: Here the components of the metric tensor fluctuate widely and without regard to each other, there will be many regions of space in which the signature of the metric changes and which no longer correspond to a singly connected Riemannian manifold. Thus Wheeler arrives at a figure of 10^{60} wormholes in the volume of space filled by a single electron. According to this elementary particles are by no means simple structures but some sort of collective modes of the froth of the vacuum.

Mrid p. 377. Dynamical theory in curved spaces I — A review of the classical and quantum action principles by Bryce S. DeWitt — Introduction:— Programme of the work falling into four parts: (1) Study and development of a number of usually neglected aspects of quantum mechanics which must be considered when attention is focussed on the non-linear features arising from the metric structure of the configuration space of a certain general class of systems; (2) Specialisation to those members of this general class in which the specific covariant geometry of the configuration continuum plays the leading role. (3) The carrying out of a reformulation of the "traditional" theory in which specialised aspects of general relativity become prominently displaced. (4) A study of interacting systems, including those of the Fermi type which possess no classical analogues, in the context of general relativity — This paper deals with

Motivation for the project not relying on recent experiments. Characteristic length for the quantum-gravitational processes is $\ell^* \left(\simeq 10^{-33}\text{ cm}\right)$ corresponding to energies of the order of 10^{19} Bev. Anyway the problem is there. Also the historical development of physics teaches a suggestive lesson in this connection, namely that the existence of any fundamental theoretical structure which is far from having been pushed to its logical mathematical conclusions is a situation which may have great potentialities [Cf. ~~[illegible]~~ pre-Lorentzian electrodynamics or pre-Lamb era quantum electrodynamics]

Misner p. 423 - Negative mass in general relativity by H. Bondi - Arguments relating to negative mass are fairly elementary & well known. Three kinds of mass: inertial, passive gravitational, and active gravitational mass. Inertial mass is defined by Newton's 2nd law (force mass independent); passive gravitational mass is the mass on which the gravitational field acts, i.e. defined by $\underline{F} = -m \operatorname{grad} V$; active gravitational mass is the source of the gravitational fields and hence is the mass that enters Poisson's eq^n & Gauss' law - 4 cases of signs of the three types of forces.

Misner p. 465 - Interaction of neutrinos and gravitational fields by D. R. Brill & J. A. Wheeler - on p. 468 remark is made that the Dirac equation in general relativity can be written in the form

$$\gamma^\alpha \nabla_\alpha \psi + \mu\psi = 0 \quad (\mu = mc/\hbar)$$

Last Section II devoted to gems i.e. an entity constituted entirely of neutrinos and held together by their mutual gravitational attraction (Also this implies creation of gravitational fields by neutrinos and the response of neutrinos to gravitational fields). - Spinor fields in general relativity [Bade & Jehle; Rev. Mod. Phys., 25, 714 (1953) for general review] - In conclusion remark that the role of spinors is still mysterious

11/10/68 Rev. Mod. Phys - 1962 - Misner p. 873 - Article by J. A. Wheeler (vol. dedicated to Pauli) on "Problems on the frontiers between general relativity & diff. geometry - Introduction: The contributions of mathematics & Physics - The unexpected richness of general relativity; only one basic principle to write Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = 8\pi T^*_{\mu\nu}$$

Radius of Universe still as far $\sim 10^{28}$ cm & [Planck's characteristic length $\sim 10^{-33}$ cm for frontiers in geometry, $(\hbar G/c^3)^{1/2}$] both as far removed as can be from the domain of everyday experimentation - Insights needed to unravel the implications of Einstein's eqns are not yet present. There is no reason that this situation will change. So geometrodynamics will be forced to evolve a pattern extraordinarily hard to find among any of the other branches of Physics. It will require a long and arduous effort almost unassisted by immediately relevant experiments. It will demand the closest sharing

and intermingling of the most modern developments of mathematics together with physical analysis of concepts and model situations at all levels – Is geometrodynamics valid at small distances? If so it must come a large part of the way towards explaining how elementary particles are built out of pure geometry. For general relativity to be of any use in discussing such a question, Einstein's equations must make sense not merely down to 10^{-13} cm, but to distances 20 orders of magnitudes smaller – Unprecedented analysis of consequences of theory required before any correlation with elementary particle physics can take place – Elementary particles as almost negligible perturbations in vacuum physics – No substitute known for Einstein's geometrodynamics – Gravitational & em. forces compared & contrasted – Daring dream of Einstein that the Universe is built out of pure geometry and nothing more, still only a vision but an inspiring one – Nature of the problems uncovered by the answers! Goal for the main problem: to discover what *is* the problem! What does one want to do with Einstein's equations? What does one want to find out? where does one want to go? – All kinds of considerations on gen. relativity – Central point: The analysis of the hidden aspects of Einstein's theory makes new demands, both in mathematics & physics.

Ibid, 1965. p. 64 – Symmetry groups in Physics – Elementary treatment meant for explic. physicists – ~~nothing wonderful~~ Until a comparable physical substructure emerges to unify our picture of the elementary particles world, a rather blind faith in the benevolence of divinely guessed at groups seems to be all we have to light the way: a shell model with imagined particles being put into unknown shells.

(A.3) (A1 & A2)

The three articles (Abstracts) I, II & III in P.R.L of 26/6/67 and 31/6/67.

I – A 6-dimensional Riemannian manifold introduced – Canonical formulation of General relativity – the quantized Friedman Universe is studied in detail – ~~while~~ ends with some speculations on the uniqueness of the state functional of the actual Universe.

(Canonical Theory)

II – ~~was~~ the manifestly covariant Theory – There exists at present no tractable pure operator language in which to base a manifestly covariant quantum theory of gravity. one must construct the theory by analogy with conventional S-matrix theory, using the c-number language of Feynman amplitudes when nothing else is available – To be read again

III – Talks of gravitons.

16/10/68 – Some more extracts from Feynman's article on the quantum theory of gravitation in Acta Phys. Polonica, 24, 1963, pp. 697–722.

My interest in it is primarily in the relation of one part of nature to another. There is a certain irrationality to any work in gravitation, so it is hard to explain why you do any of it. This irrationality is shown also in the strange gadgets of Prof. Weber, in the absurd creations of Prof. Wheeler & other such things. It is therefore clear that the problem we are working on is not the correct problem; the correct problem is what determines the size of gravitation! But since I am among equally irrational men I won't be criticized I hope for the fact that there is no possible, practical reason for making these calculations. ..... At the suggestion of Gellmann I looked at the theory of Yang–Mills with zero mass which is in difficulty or like the gravity theory in two diseases, if one is cured they both can be cured; Yang–Mills theory is easier to compute with than the gravity theory, e.g. to calculate the diagram Fig. 4b (p. 707) the Yang–Mills case took me about a day; to calculate the diagram in the case of gravitation I tried again and again but never was able to do it and it was finally put on the computing machine, not the arithmetic, but the algebra of the thing was (I did the integrals later). Thus this is perhaps a historical example of algebra having been done on a computing machine for the first time. – I don't understand these results at the moment, and I therefore do not see clearly in that this method of quantization (using diagrams of loops & trees & soon) can be obviously and evidently carried on to the next order. – I can however relax from the problem & retire on the claim that the number is so small, and that the thing is now really irrational, if it was not irrational before. The trouble is present in the Yang–Mills theory also & I have incidentally discovered a interesting connection which is of very great interest and importance in the meson theories and so on. And I am stuck to have to continue this investigation, and of course you all appreciate that this is the secret reason for doing any work, no matter how absurd and irrational and academic it looks: we all realise that no matter how small a thing is, if it has physical interest and is thought about carefully enough, you are bound to think of something that is good for something else.

Further extracts from the discussion: Reply to a question of Ivanenko re. the transmutation of two particles into graviton: – I never went to energies more than a billion–billion BeV & then the cross-sections of any of these processes are infinitesimal — the thing that is the analogue of $e^2/\hbar c$ in electricity which is 1/137 is non-existent in gravitation; it depends on the problem; this is because of the dimensions of G. So if E be the energy of some process,

than if you take $GE^2/\hbar c$ you get an equivalent to this $e^2/\hbar c$. But this would result in the conclusion that to get an appreciable effect you've got to go to ridiculous energies. So either you either have a ridiculously small effect or a ridiculous energy.

18/11/68 Feynman takes the Lagrangian of Einstein and puts $g_{\mu\nu} = \delta_{\mu\nu} + \chi h_{\mu\nu}$ getting a big mess

$$\mathcal{L} = \frac{1}{\chi^2}\int R\sqrt{g}\,dt + \frac{1}{2}\int\left(\sqrt{g}\,g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - m^2\sqrt{g}\,\varphi^2\right)d\tau \qquad (1)$$

Substituting and expanding & simplifying by using notation $\bar{x}_{\mu\nu} \equiv \frac{1}{2}(x_{\mu\nu} + x_{\nu\mu} - \delta_{\mu\nu}x_{\sigma\sigma})$

$$\mathcal{L} = \int\left(h_{\mu\nu,\sigma}\bar{h}_{\mu\nu,\sigma} - 2\bar{h}_{\mu\sigma,\sigma}\bar{h}_{\mu\sigma,\sigma}\right) + \frac{1}{2}\int\varphi_{,\mu}^2 - m^2\varphi^2)\,d\tau$$

$$+\chi\int\left(\bar{h}_{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} - m^2\tfrac{1}{2}h_{\sigma\sigma}\varphi^2\right) + \chi\int "hh\hbar" + \chi^2\int "hh\varphi\varphi" + \text{etc}$$

& uses notions of loops & trees and all sorts of nonsense —

[The effect of gravitation on the H-atom is to shift the phase by 43 seconds of phase in every hundred times the life-time of the Universe! An atom made purely of gravitation, let us say, two neutrons held together by gravity has a Bohr orbit of 10^8 light years. The energy of this system is 10^{-70} rydbergs. I wish to discuss here the possibility of calculating the Lamb correction to this thing, an energy, of the order of 10^{-120} — carrying out the perturbations to this limit - I am investigating the subject despite the real difficulty that there are no experiments. Therefore there is no real challenge to compute true, physical situations. And so I make up believe that there were experiments .... The plan of the attack is a succession of increasingly complex physical problems .. Also I decided not to investigate what I would call familiar difficulties - for eg. non-linearity is not an unfamiliar difficulty; the theory for eg. of the spin ½ particles interacting with the e.m.f has a coupling term $\bar{\psi}A\psi$ which involves three fields & is therefore non-linear; that is not at new thing at all — Again the question of divergence was not investigated because for eg. quantum electrodynamics also diverges — free graviton with no source called an honest graviton — The proof that the response to a gradient potential is zero required that the other particle that was interacting was an honest graviton — what happens if there are two or more loops — I did investigate this as well before I came here & I haven't had enough time to investigate this to my own satisfaction. Preliminary investigations show that I cannot claim that this method of quantization can be obviously and evidently carried out to the next order. In short therefore, we are still not sure of the radiative corrections to the radiative corrections to the Lamb shift, the uncertainty

lies in energies of the magnitude of 10^{-255} rydbergs.]

(20/11/68) Again from Feynman's article

"So I am stuck to have to continue this investigation, and of course you will all appreciate that this is the secret reason for doing any work, no matter how absurd and irrational and academic it looks; we all realise that no matter how small a thing is, if it has physical interest and is thought about carefully enough, you're bound to think of something that is good for something else.

(1) – nothing worth noting.

(2) –

(3) — ✓ — above remarks of Feynman

(4) Sci. American, 1957 February, p. 99 – Article by D. Sciama on "Inertia" – Discuss Mach's Principle reviving the famous Philosopher Berkeley's idea that fixed stars are responsible for inertia – Einstein using Mach's Principle & Principle of Equivalence – the argument that Newton's method work is not a refutation of Mach's Principle – It follows from this principle that G is related to mean density of matter in space, in fact, at distances greater than those to which the 200"-telescope can penetrate (for eg. Sun's contribution to local inertia is only about one hundred-millionth of the total from the Universe) – This ~~suggests~~ warns that the Universe may play a hitherto unsuspected role in other phenomena – for eg. Charge on an electron e appears independent of environment, and indeed classed as a fundamental constant of nature, but this property, like inertia, may also be the result of some long-range interaction – If atomic properties are thus determined, we have the dual situation: distant matter influencing local phenomena, and local phenomena giving information about distant matter.

(5) In addition to summary in M.R – Supports Wigner's argument that there is ~~a~~ no conceptual difficulty in uniting Special relativity & Quantum Mechanics, but that there is hardly any ground common ground between G.R & Q.M (Note: Wigner also states that the first 1st part of above statement does not mean that we have a satisfactory Lorentz-covariant quantum theory which is perfect and well-defined in the mathematical point of view) [Mention this paper of Utiyama also in the Introduction]

(4) – From the Introduction – The assumption made in the paper may be brushed aside by saying that it is possible to treat gravitation as just another particle field, thus avoiding the idea of a curved space-time altogether. This argument can be answered by saying that if such a flat-space, essentially linear and perturbational treatment of gravitation is adequate, then the

-smallness of the coupling constant removes this science from relevance to elementary particles; hence, the greatest interest in the subject depends on the possibility, however remote, that the non-linearity is essential, and the only understanding of non-linear effects has come from Einsteins geometrical interpretation. Re. the objection that the Klein-Gordon & Dirac eqns may be easily generalised, this argument is totally irrelevant - for the modern theories of elementary particles, both field theory and the phenomenological treatment, are not studies in diff. eqns. The construction of a physical theory and the interpretation in particular rest instead on the concepts of energy, momentum and spin, whose existence is due to invariance principles - more specifically, the principle of invariance under the inhomogeneous Lorentz group (Poincaré group ?). From this pt. of view the K-G eqn is interesting because the solns form the basis for an irreducible unitary representation of that group.

The inhomogeneous Lorentz group comes to the fore as the group of motions of Minkowski space. The absence of groups of motions in more general Riemannian spaces is a formidable obstacle to the construction of physical models. However, for Riemann. spaces of constant curvature, we have a group of motions, although different from that of flat space, that has the same number of parameters. On such a space, we may define energy, momentum, mass and spin. We may study the irreducible representations, elementary particles & their interactions. We study this case only, identify a group, make a preliminary study of the irreducible unitary reps, and show how intimately some of these are related to the reps of the L-G. — Ref. to Eisenhart's book on "Continuous groups of transformations") stating that a 4-dimnl Riemannian space may admit a continuous group of motions with up to ten essential parameters. The maximum number is only realised only for a space of constant curvature i.e. where curvature tensor takes the form $R_{\mu\nu\lambda\rho} = C(g_{\mu\nu}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})$ (C = a const.)

(6) This is the article by J. Rayski supposing unified field theories — [The remark under (6) in M.R. appears does not appear to have been mentioned in this article; anyway the remark is good enough for being put in the Introduction for original attempts appeared justified when only gravitational & e.m. fields were known. However, in view of the discovery of other fields (nuclear fields, meson fields) it is little hope to construct a unified theory of all these fields, so that picking up only two of them (resp. gr. & e.m.f.) does not seem to be of interest nowadays. & also these theories are formulated in an old-fashioned way. — The author introduces a modernized exposition, but sticks to gravito-electromagnetism (since gravitons & photons are the only two massless bosons) as not being unimportant — claims that his new

5-dimensional theory throws light on many things viz (i) interpretation of charge-conjugation as inversion of the fifth coordinate may contribute to a future better understanding of the problem of weak interactions with their characteristic property of violating the conservation of parity. (ii) interpretation of the puzzle of isospin — makes some general remarks about this fifth dimension etc [It is difficult to understand how this particular hyperspace might field theory can throw light on weak and strong interactions etc]

21/11/68

(7) not available - missing from the library - Phys. Letters is from Netherlands & is diff from Phys. Rev. Letters.

(8) mentions about usual procedure of quantisation viz. obtaining P.B's in classical theory → hermitian operators in quantum theory — when we turn to GTR, the preferred subgroup of the canonical group is no longer the Poincaré group (inhom. L. G) but rather the group of general coordinate transformations (ie Einstein group) The usual programme of quantising does not work because the Einstein group is a function group and in view of the fact that the Einstein group is simple, it appears that there exists no unique procedure for constructing a quantum theory of relativity — Talks of truncated GRT consisting of some of Einsteins equations satisfying suitable boundary conditions at ∞ so as to enable a discussion of gravitational radiation. In this truncated theory one has the generalised Bondi-Metzner (GBM) group [Ref. R. Sachs - Phys. Rev. 128, 2851 (1962)] which is remarkably similar to the Poincaré group, although considerably larger; it is an infinite group. The Poincaré group is a subgroup of the GBM group, but, however, not a normal subgroup. GBM has two abelian normal subgroups viz (T) (4-dimensional group associated with translations) & an ∞-dimensional group (ST) called the supertranslation of which (T) is a normal subgroup. The proper supernormal translations (PST) ≡ (ST)/T are not normal subgroups of the GBM. The similarly with the Poincaré group arises from the property GBM/ST = L(hom. L. G). Finally, it is important to consider the group I = GBM/T, which is an ∞-group & it has an ∞-abelian normal subgroup isomorphic to PST, and I/PST = L. — Suggests that I must be considered suitable as the internal-symmetry group & it is in consonance with conclusions of O'Raifeartaigh [Phys. Rev. letters. 14, 575, 1965]. Also I is not obtained by semi-empirical arguments. It is a consequence of the well-established theory of general relativity and reflects the heretofore ignored gravitational properties of elementary particles. Thus even if I should be prove to be incorrect, it would be important to understand why!

[Two articles by O'Raifeartaigh appear in Phys. Rev. Letters 14, p. 575 and 14 p. 332 (1965) which both of which gives references to E. C. G. Sudarshan & the latter refers to some Hooker - Marcer - Harish-Chandra theorem given in Jacobson's book on Lie Algebras, 1962, pp. 24-25. Anyway neither paper makes clear the notion of an internal symmetry group].

(9) Summary: It is possible to quantise most classical field theories by identifying the group of canonical transformations which maintain the covariance properties with a group of unitary transformations in Hilbert space which has the same commutator algebra. The commutators among the canonical field variables are equal to the Dirac delta function times a factor which may be zero. But in the GTR the classical group of canonical transformations which maintain the covariance properties of the theory has an invariance sub-group. The ambiguities thus introduced by the usual process of quantisation can be avoided by the use of the Dirac quantisation procedure for theories with constraints. We establish an analogy between classical Dirac brackets and commutators, and fix an intrinsic coordinate system. This choice of local intrinsic coordinate conditions leads to the commutators among the canonical field variables of the GTR which depend upon the Dirac delta function and its first seven derivatives.

[References to Dirac (i) Can. J. Math. 2, 129 (1950); 3, 1 (1951)

(ii) Proc. Roy. Soc. A. 246, 326 and 333 (1958)

(iii) Phys. Rev. 114, 924 (1959)

Also to Komar, Pirani, Peres, Bergmann etc. etc.]

(10) Introduction: In addition to the lengthy treatments in M R: Logical structure of physics is often anti-parallel to its historical development (modern field theorists tell us for example) Maxwell's & Einstein's equations are the starting point of the quantum theory of photons & gravitons). For the purposes of this article, we suggest the reader is requested to forget all he knows of electrodynamics and general relativity and instead to take his starting point the Lorentz invariance of the S-matrix calculated by Feynman-Dyson perturbation theory. That is, we assume the S-matrix to be given by

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int_{-\infty}^{\infty} dt_1 \ldots dt_n \, T\{H'(t_1) \ldots H'(t_n)\} \qquad \ldots (1{\cdot}1)$$

with $H'(t)$ the interaction Hamiltonian in the interaction representation

$$H'(t) = \exp(\text{[illegible]}) \exp(iH^F t) H' \exp(-iH^F t) \quad \ldots (1{\cdot}2)$$

where H^f is the free particle interaction Hamiltonian and H' the interaction. our fundamental requirement on the form $H'(t)$ is that (1·1) must yield a Lorentz-invariant S-matrix.

The power of this requirement is only now beginning to be appreciated. There are many indications [Weinberg, Phys Rev. 133, B·1318 (1964) for case $m \neq 0$, and ibid 134, B·882 (1964) for case $m = 0$] (though as yet no careful proof) that it yields all the results usually associated with local field theories, including existence of anti-particles, crossing-symmetry (?), spin and statistics, CPT, the Feynman rules (?) etc. The purpose of this article is to explore the consequences of Lorentz invariance in perturbation theory for the special case of zero mass & integer spin.

Result:- Maxwell's theory and Einstein's theory are essentially the unique Lorentz-invariant theories of massless particles with spin $j = 1$ & $j = 2$. Essentially only means that the conserved current g^μ and $g^{\mu\nu}$ in which p & q are coupled need not be precisely equal to the charge current J^μ and stress-energy tensor $\theta^{\mu\nu}$. Also no theory for $j \geq 2$ yielding universe agrees with macroscopic fact.

[Ref. to Gupta's article in Recent Developments
Schwinger: Nuovo Cimento 30, 278 (1963)]

mathematical works of Utiyama; Einstein & Thirring theories as alternatives.

§ 5. Lorentz invariance & Feynman rules:

on p. B·999 - refers to S. N. Gupta - Proc. Phys. Soc. A·65, 608 (1952) as suggesting that there is only one choice of a conserved $\theta^{\mu\nu}$ which makes eqns (7·25) equivalent to Einstein's non-linear eqns & refers to a private communication by Feynman showing that this is the only choice which works.

22/11/68

(11) - (13) - not available

Extracts from the book by R. H. Dicke on the theoretical significance of experimental relativity:

(Documents of Modern Physics Series)

Preface: A well known physicist once remarked to me that Einstein's general relativity was such a beautiful theory that it was a shame that there were so few experiments. - Because of the weakness of gravitation, results are far easier to obtain by calculation than by measurement. The experimentalist is unable to

complement and guide the theorist in the manner that occurs in the other hard sciences of Physics — It would be a shame for a dozen of very able theorists to devote a second half century to developing the results of general relativity only to discover eventually that the theory is defective, that some basic assumption is incorrect.— Philosophic considerations of Berkeley & Mach used — two of the cardinal principles of ~~gravitation~~ general relativity are the strong equivalence principle, and the principle of a purely geometrical description of gravitation — In this book the second principle is demoted from the central position now occupied to a secondary position; whether this is satisfactory or not is a question of aesthetics — This demotion makes possible ~~a~~ an enormous class of possible relativistic theories of gravitation, but the expts. discussed make possible only Einstein's theory and the scalar zero-mass field theory — we seem to be very far from a situation requiring gravitons and the quantisation of the gravitational field — Berkeley's view that the motion of a single test particle in an empty physical space (a void without properties) is a meaningless concept, as also that of an acceleration caused by the force — Mach's view that inertial forces were action-at-a-distance ~~forces~~ dependent ~~on~~ acceleration dependent forces produced by interactions with the distant matter of the universe; the modern interpretation would be to interpret inertial forces as acceleration dependent interactions of a particle with the field generated by the distant matter in the Universe.

Introduction :— The situation re. expts in gen. rel. activity is almost orthogonal to that in high energy physics ~~experiments~~ in the 30's & 40's that gen. rel. was of little importance to the physicist and that cosmology is of no concern to the physicist — this is too provincial point of view; the walls of the physicist's laboratory are penetrated by neutrinos, probably by gravitons and possibly by scalarons generated by distant matter in the universe in ancient times. Also the structure of the elementary particles, his stock and trade, may be dominated at small distances by gravitational effects — Many cosmological effects if they could be observed, would give information about the most fundamental problems of physics.— The Galaxy as a giant laboratory in which many million experiments are being carried out simultaneously on an enormous scale; the physicist may want to look around carefully to see if by some fluke ~~exp~~ some expts. is just the same as the one that he wanted to perform in his laboratory himself

§ 1. Null experiments — Eötvös's experiment (measurement showing with great precision that all bodies fall with the same acceleration); null result of this expt. a necessary condition for GTR being valid — Expts. by the Princeton group of comparing the gravitational acceleration toward the sun of small gold and aluminium ~~weights~~ bobbing ~~them~~ to be the same with an accuracy of 1 in 10^{11} — No experiment can provide a sufficient condition for a theory to be valid, as all experiments have only a limited accuracy — Strong equivalence principle on which Einstein's theory is based, and the weak equivalence principle supported by Eötvös's expt — The latter states that only that the local

gravitational acceleration is substantially independent of the composition and structure of the matter being accelerated. The former requires that the laws of physics, expressed in a coordinate system, locally inertial shall take on some standard form and have some standard numerical content. i.e that physical laws are correctly described in gravity free space by the usual Lorentz-invariant formalism, and this implies that for the more general situation where gravitation is present, the formalism should reduce locally to this standard invariant form (with local Minkowski coordinate emitters). It is well known that this interpretation of the equivalence principle, plus the assumption of general covariance is most of what is needed to generate Einstein's general relativity (coincides with treatment in Pauli's book) — characteristic fundamental gravitational mass $(\hbar C/G)^{1/2}$ ever and fine structure constant & other constants. One of the assumptions of the strong equivalence principle is that these dimensionless" constants"are truly constant in coordinate independent — In recent years it has frequently been suggested that the gravitational pull on an anti-particle might be negative. This suggestion ignores the fact that it is incompatible with the requirement of GTR. Schiff's simple argument was (b.7) — The fantastically small Fermi & gravitational coupling constants $\simeq 10^{-13}$ & 10^{-40} respectively — The weak principle of equivalence is supported by Eotvos expt, and except for the question of invariance of the above two constants, the strong principle also is supported by this experiment — The Princeton Expt performed by Roll, Krotkov & Dicke re. accn of Sol & Al. towards the Sun — Null expt of Hughes, Robinson & Beltrandopes on spatial isotropy and independently by Drever (called Hughes - Drever expt) — It is well known as a result of the investigations of Rosen, Gupta, Feynman, and Thirring that the gravitational field can be treated within the framework of ordinary Lorentz invariant field theory as a zero mass, Charyaless tensor field. [Rosen: 1960; Phys. Rev. 57, 147; Gupta: 1957, Rev. Mod. Phys. 29, 334; Feynmanna: 1957, Chapel Hill conference; Thirring: 1961, Annals of Physics, 16, 96-117]. These results interpret the gravitational field as an ordinary tensor interaction and are non-trivial since they open up possibilities of other types of long range interactions. Poincaré's interpretation of geometry as "geo-metry" (i.e first half of the word relates to the space & the second half refers to its measurement, tho eg. Riemannian space may be considered flat and the metre stick distorted — "Proper rods & clocks" can be considered as those adjusted to agree always with the "invariant length & time units" $(G\hbar/c^3)^{1/2}$ and $(G\hbar/c^5)^{1/2}$ — Presence of two tensor fields is excluded by the Hughes-Drever expt (long meaningless explanation) — Ether Drift experiments: — Those like the Michelson-Morley expts — the most interesting type of null experiment described (b.23) being an attempt made (about 1962) to measure the redshift using the Mössbauer effect just after this was discovered (long description).

§2 — Three famous tests of GTR. — (a) gravitational redshift — is nothing but a test of energy-conservation and mass-energy equivalence; explanation of two accurate tests by several persons.

(b) Gravitational deflection of light – uncertain to at least by 10% & perhaps as much as 20% (c) The perihelion rotation of Mercury – uncertain

Cosmic experiments – observations of geophysical and astrophysical nature – impossibility of using these types of data to "prove" such a theory

Appendix I – Experimental Tests of Mach's Principle :- The Hughes-Drever & other experiments confirm Mach's Principle (ie the inertial forces experienced in an accelerated laboratory are gravitational (by R.H.Dicke) having their origin in the ~~matter~~ distant matter of the Universe, accelerated relative to the laboratory). These experiments show, with great precision, and in accordance with Mach's Principle that inertial anisotropy effects are universal, the same for all particles.

Appendix 2 – by R.H.Dicke – Mach's Principle and invariance under transformation of units :- A gravitational theory compatible with Mach's Principle recently published by Brans and Dicke, characterised by a gravitational field of the Jordan type (Born Schwerkraft und Weltall), tensor plus scalar field. By change of units (coord-dep. ~~trans~~ trans of units), the theory can be put in the form of Einsteins field equations, and the scalar field then appears as a "matter-field". But formally and in its physical interpretation, the theory is different from general relativity – A units transformation can be used to redefine Riemannian geometry in such a way that the resulting geometry is flat. Invariance under transformations of units – $g_{ij} \to \lambda g_{ij}$, $g^{ij} \to \lambda^{-1} g^{ij}$, $\hbar$ & c constants, $m \to \lambda^{-1/2} m$, ~~where~~ where λ is assumed a definite function of the scalar of the Brans-Dicke formalism [Phys. Rev. (Ref.1) 124 (1961)].

Mach's Principle, expressed in transformed units :- In Ref.1, the gravitational coupling constant G was interpreted not as a fait accompli presented to us by nature, and (4) ~~–~~ $G m^2/\hbar c \approx 10^{-41}$ unrelated to other parts of physics, but rather as a number determined, coincident with the requirements of Mach's Principle, by the total mass distribution in the Universe. Being a dimensionless number, it is invariant under transformation of units. In Ref.1, $\hbar$, c & m were constant by definition and G constituted dependent, while here we take G, $\hbar$, c constant and all others (m included) varying with position being functions of φ – A long four page ~~rubbish~~ – Summary and conclusion – Eqn (4) above may be interpreted as small rest particle mass m is reduced drastically by interaction with the fields generated by the enormous amounts of matter in the Universe – the relation $GM/Rc^2 \approx 1$ is understandable (M = mass of Universe out to visible limits & R the Hubble radius which is a measure of the certain

this visible horizon). Measured in these units, the masses of elementary particles adjust themselves, through the scalar field generated by all the matter, in such a way that M/R stays constant and of the order of magnitude of $G^{-1}c^2$.

Appendix 3 – R. H. Dicke – Long-range scalar interaction – The long range scalar field associated with a neutral, massless boson has been generally considered to be non-existent. This belief is based on the lack of overt effects, observed in the laboratory, from such a field. It is shown here that if this long-range interaction were to exist, it would be necessarily weak. The physical reason for this is the large contribution, having its origin in the enormous amounts of matter at great distances in the Universe, to the magnitude of the scalar. By comparison, the contribution of local matter is miniscule, leading to a weak interaction of about the same strength as gravitation. Furthermore, it is shown that such an interaction, in its effects, would be very similar to gravitation and could be distinguished only with difficulty. It is concluded that there is not yet a compelling observation which could be used to exclude the long-range scalar interaction. – If it be assumed that nature abhors a field more complicated than a tensor in an elementary particle of spin greater than 2, then only three types of fields remain viz. scalar, vector and tensor. Vector fields like the e.m. field are important for cosmology in view of vanishing of the E- and H-fields consequent on isotropy of space. – If one were to have some small faith in the proposition that nature is not capricious, that the physical world is ultimately simple, then the simplest of the three fields, the scalar field might be expected to exist and to play an important role in the universe. – In particular, if this field exists, the extremely small value of $(G m_e^2/\hbar c) \sim 10^{-40}$ is recognized as the effect of the enormous amount of matter in the universe generating a scalar field which acts to depress the value of m_e, the mass of an elementary particle.

Appendix 4 – R. H. Dicke – Field theories of gravitation.

Only gravitation (weakest interaction) is singled out for the honor of geometrical interpretation – The work of Rosen ... – Driving out others can be treated as a field within the framework of Lorentz invariant theory, but these treatments are only formal, the metric tensor of the Minkowski metric being unobservable – The assumptions made in obtaining possible types of field theories of gravitation here are listed below: –

(1) Geometrical points are to be associated with physical events

(2) only geometrical properties of space introduced à priori are those of a 4-dimnl space with an arbitrary Coord. system varying sufficiently smoothly (?) to characterise a differentiable manifold. There shall be neither a metric nor affine connection.

(3) Space will be assumed to have no intrinsic (objective) properties beyond those of matter contained in the space. The dynamical eqns will be constructed in generally covariant form, and they will contain no elements referring to space except the arbitrary Coord. system (where the coords refer to physical events)

(4) Dynamical eqns will be obtained from an invariant action principle.

(5) Large number of bosons of the same kind can be characterised in their interaction by a tensor (ie a scalar, vector or tensor field)

(6) Final assumption, which cannot be easily justified viz. Nature abhors a complicated situation. It will be assumed that if a linear superposition of interactions is possible, it will occur in preference to a non-linear one ie Nature likes things as simple as possible.

After nearly 6 pages of nonsense, the author concludes that there is an enormous range of theories, but the stringent conditions of the Eotvos & Hughes-Drever expts. appear to narrow down the class to some combination of a single tensor field and a (?) scalar field interacting only with the tensor field, or else with the tensor field and all particles through the mass terms. Finally, he says that it must again be emphasised that the introduction of a metrical geometry always require an arbitrary assumption (as if the assumptions above at least No. 6 are not arbitrary). Only when the gravitational effects are all due to a single tensor field (Einstein's theory) can this assumption be made to appear unique, the one choice of metric tensor being prettier (?) than all the tensors.

[Instead of all this I know the assumption of prettiness would lead to Einstein's theory!

Appendix 5 – R. H. Dicke – Cosmology, Mach's Principle and Relativity.

Cosmology an enchanting science; original roots in philosophy; lifted above the level of conjecture thanks to relativity theory; principles of GTR applicable to cosmological & verifiable by cosmological expts – Bishop Berkeley's (obviously the idiot who is the author's ideal) insistence that the only meaningful motion of a body was motion relative to other matter is the relativity principle. This idea recurs in writings of Mach, and in the theory of relativity as developed by Einstein. Mach asserted that, assuming the relativity principle, the inertial forces

origin in the distant matter in the Universe, and this has great significance for cosmology. and the physicist cannot also ignore cosmology – Scalar, vector and tensor fields = Cosmic fields; the long range Tensor field, through the acceleration dependent force which it generates, is a cosmic force par excellence. for, its influence, propagated from distant parts of the universe, permeates insidiously the thickest and best shielded walls of the physicists inner sanctum, affecting all his measurements; the scalar field of G is assumed to be non-existent and for laboratory physics its observational effects are almost non-existent – Cosmic fields and geometry: Dirac was the first to suggest a connection between the mass-content of the universe, and the gravitational coupling constant decreasing with time – Einstein's "general relativity" is rather a special theory for which the scalar field is assumed not to exist. This has been emphasised by Synge in his new book (Relativity, the general theory, 1960) – some remarks about Einstein's theory not being in conformity with Mach's Principle, and Synge being correct – Einstein's theory not being a relativity theory (!) – Cosmic fields and cosmology I – discussion of the Einstein field eqns $R_{ij} - \frac{1}{2} g_{ij} R + \Lambda g_{ij} = - \frac{8\pi G}{c^4} T_{ij}$, the third term on the left being the cosmological term – argument for and against dropping this term – Steady state cosmology of Hoyle, Bondi & Gold dismissed as inconsistent with fundamental conservation laws of energy and momentum.

Appendix 6 – R. H. Dicke – Significance of spatial isotropy – Question of the fine structure constant $\alpha = e^2/\hbar c$ being variable and depending on the distribution of mass in the Universe is considered. Using the observed structure independance of gravitational acceleration, local isotropy of space, and general covariance, it seems variability of α is to be ruled out

Appendix 7 – C. Brans and R. H. Dicke – Mach's Principle and a relativistic theory of gravitation. The role of Mach's Principle in physics is discussed in relation to the equivalence principle. The difficulties encountered in attempting to incorporate Mach's Principle into general relativity are discussed. A modified relativistic theory of gravitation, apparently compatible with Mach's Principle is developed. Introduction – Sciama's result based on Mach's Principle that G is related to the mass distribution in a uniformly expanding universe as $GM/Rc^2 \sim 1$ – Dirac's result goes well beyond the empirical data on which it is based – R. H. D has shown that Dirac's results are not in accord with astrophysical observations examined in the light of modern stellar evolutionary theory – A theory of gravitation based on a scalar field in Riemannian geometry: A generalisation of general relativity – 3 pages of nonsense. – Weak field approximation – Another 2 pages – Static spherically symmetric field about a point mass – Another 2 pages – Mach's Principle in consonance with the new theory – Cosmology: 4 pages.

Appendix 8 - Lee-Yang vector field and isotropy of the Universe - Conclusion that the Lee-Yang field analogous to an e.m.f does not exist

Appendix 9 - The Earth and Cosmology - The earth may be affected by the distant matter of the universe through a long-range interaction. - Physical framework - The tensor field already provides us with such enquiringly beautiful mathematical difficulties that higher rank tensors should be forbidden by fiat - Advocates existence of the scalar field as the only one of the three tensors which is simple, [illegible] and eliminates several deficiencies in general relativity theory W.r.t. Mach's Principle & to bring the theory in accord with that Principle - Gives eleven (11) Chief results relating to properties of the long range scalar interaction (p. 104) - Cosmology and the scalar field :- Choice of scalar particularly significant for cosmology - Thermal history of the Earth - The Earth's composition - Suffers [illegible] of fundamental strength - 8 pages in all devoted to this - It must be emphasised that the foregoing discussion has a strong conjectural element ; it cannot be otherwise until we know much more than we now do about the composition of the earth and its internal temp. distn. - The Earth's magnetic field - Surface temperature of the Earth - What is badly needed to raise this discussion above the level of conjecture is a good demonstration that G has indeed been slowly decreasing.

Appendix 10 - Implications for cosmology of stellar and galactic evolution rates.

Mach's that there may be important space-curvature factors to determine the structure of particles & refers to Landau & Wheeler's article (on neutrinos, [illegible] etc) - Stellar evolution - Hertzsprung-Russel diagram - Age of stars - Galactic evolution - Evolutionary age of the Galaxy - The age of uranium - Evolutionary age of the universe - Cosmological Considerations - Summary & conclusions

Appendix 11 - Dating the Galaxy by uranium decay.

Appendix 12. - Dirac's cosmology and the dating of meteorites.

Makes the assumption somewhere that nature is capricious but not malicious.

18/12/68

(1) Phys. Rev. Letters · 8/7/68 - p. 106 - New formulation of general relativity by E. Gaultt (Ref. to Dicke's book)

(2) " 19/8/68 - p. 562 - Particle creation in expanding Universe.

References in (1) R.H. Dicke's book.

(2) P.G. Bergmann - Handbuch der Physik, 1962, Vol 4, p. 203

[3] J. Weber - General relativity & gravitational waves (4) Anderson's book

(5) S. Weinberg - 1964 Brandeis Lecture Notes (Prentice Hall N.J. 1965, Vol 2, p. 405

(6) J.N. Goldberg - Gravitation chs (Wiley ed L. Witten (John Wiley & Sons, 1962) - Book, Chap III

(7) Whitrow & Morduch - Nature, 188, 780 (1969 ?)

(8) Vistas in Astronomy Ed. A. Beers (Pergamon), Vol. 6, p. 1

(9) Schiff - Journeys in Science Ed. D.L. Arm (1967) Chap. 6, p. 148

(10) W. Davidson & J.V. Narlikar, Rept. Rep. Prog. Phys. 29, 539 (1966)

References in (2) → (1) Refers to general relativistic Dirac eqn

$$\gamma^K \nabla_K \psi = \mu \psi$$ in Bargmann's work

[Bargmann, Sitz. Deut. Akad. Wiss. Berlin
Math - Nature, Kl, 1932, 346]

(2) H. Alfven, Rev. Mod. Phys 37, 652 (1965);

O. Klein, Nature, 211, 1337 (1967).

20|12|68 - Saw the first article in Phys. Rev. Issue of 25|7|68 on "Quasars & Quarks" - Since quarks have not so far been detected in Lab. Expts, suggests possibility of finding them in Quasars (ie Galaxies connected with radio emission).

23|12|68 - Ref. in Casiorowicz's bibliography (p. 607) to article by R.P. Feynman, Phys. Rev. 76, 749, 769 (1949) (for Sum and Integrals). [Saw & found nothing for Sum or Integral]

(i) Find a unit vector normal to the planes of A and B. [Ans: $(\hat{i}-3\hat{j}-2\hat{k})/\sqrt{14}$]

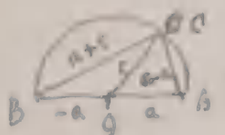
(ii) Find the area of the Δ^{le} with A and B as sides [Ans: $\sqrt{26}$]

(10) Given $A_1 = 4\hat{i}$, $A_2 = 3\hat{j}$, $A_3 = 2\hat{k}$, find the reciprocal set of vectors [Ans: $\hat{i}/4$, $\hat{j}/3$, $\hat{k}/2$]

(11) " $A_1 = 2\hat{i}-\hat{j}+3\hat{k}$, $A_2 = -\hat{i}+3\hat{j}+2\hat{k}$, $A_3 = \hat{i}+\hat{j}-2\hat{k}$, find reciprocal set

[Ans: $(9\hat{i}-\hat{j}+4\hat{k})/31$, $(-\hat{i}+7\hat{j}+3\hat{k})/31$, $(12\hat{i}+9\hat{j}-5\hat{k})/31$]

(12) Angle in a semi-ckt @ a rt $\angle^{le}$



$\vec{BC}\cdot\vec{AC} = (\vec{a}+\vec{c})\cdot(\vec{c}-\vec{a}) = c^2-a^2 = 0$ (∵ c = a)

(13) Soln to (3)

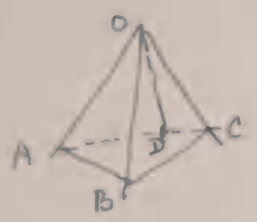
$c = b-a$; $c\cdot c = (b-a)\cdot(b-a) = b^2+a^2-2a\cdot b$

$\therefore$ that $c^2 = a^2+b^2-2ab\cos\theta$.

(13) Vector $\vec{a}$, $\vec{b}$, $\vec{c}$, $\vec{d}$ are coplanar, prove that $(a\times b)\times(c\times d) = 0$

(14) If $a+b+c=0$, prove that $a\times b = b\times c = c\times a$ and interpret result trigonometrically.

(4) The sum of two face angles of a trihedral angle is greater than the third



Let AOC be the largest of the 3 face angles. On face AOC, face AOC, draw $OD = OD$ and $\angle AOD = \angle AOB$, & draw a plane thro' B & D cutting the faces in ΔABC

$\Delta AOB \equiv \Delta AOD$ & $\therefore AB = AD$ & $AB + BC > AC\ (= AD + DC)$.

$\therefore BC > DC$. In Δ^s BOC & COD, OC is common, $OB = OD$ and $BC > DC$

$\therefore \angle BOC > \angle DOC \quad \therefore \angle AOB + \angle BOC > \angle AOD + \angle DOC$ or $\angle AOC$.

(5) Some Old Tripos Work

Four points A, B, C, D in space are joined (in order) four lines which are then bisected. Show that the four lines joining (in order) the four mid-points of the first four lines form a $\square^m$

[Pass a plane thro' ADB and a plane β through BCD, α and β intersecting in BD. Line joining mid-pts of BC & CD is $\parallel^l$ & $= \frac{1}{2}$ of BD. Same line joining mid-pts of AB & AD is $\parallel^l$ & $= \frac{1}{2}$ of BD. Hence the result]

Vector Analysis (by J. B. Marsh - Advanced book)

(1) Consider a unit cube with one corner at the origin & three adjacent sides along the three axes of a rect. coord. system. Find the vectors which represent the diagonals of the cube.

(2) Let $\underline{A}$ be an arbitrary vector, and let $\underline{e}$ be a unit vector in some specified fixed direction. Show that

$$\underline{A} = \underline{e}(\underline{A}\cdot\underline{e}) + \underline{e} \times (\underline{A} \times \underline{e})$$

What is the geometrical significance of each of the two terms of the expansion?

(by H. E. Newell)

(1) Show that a necessary & sufficient condition that three distinct pts A, B, C be collinear is that $l\underline{A} + m\underline{B} + n\underline{C} = 0$, where $l + m + n = 0$

[Hint: Points A, B, C are collinear if & only if C, $-A$ and $C - B$ are $\parallel^l$]

(2) Show by vector means that the medians of a Δ^h meet in A, B, C intersect in the common pt. $\frac{A+B+C}{3}$.

(3) $A = B - C$. From $A^2 = (B-C)^2$ prove the law of cosines

(4) Show by vector means that the diagonals of a $\square^m$ are $\perp^r$ if & only if the $\square^m$ is a rhombus or a square.

(5) Show that: (i) $[(A \times B) \times A] \cdot A = 0$, (ii) $[(A \times B) \times A] \cdot B = A^2 B^2 \sin^2 \theta$

(6) Show that $A \times B + B \times C + C \times A$ is $\perp^r$ to plane of the points A, B, C

(7) Show that $(A \times B) \cdot (B \times C) \times (C \times A) = (A \times B \cdot C)^2$

(8) Prove $A \times B = \begin{vmatrix} i & j & k \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$

(9) If $A = i + j - k$, $B = i - j + 2k$, $C = 2i + j + 2k$, ~~show that~~

(d) Solve by determinants $4x - y + 4z = 2,\ 2x + 3y + 5z = 2,\ 7x - 2y + 6z = 5$

$$x = \frac{\begin{vmatrix} 2 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & -1 & 4 \\ 2 & 3 & 5 \\ 7 & -2 & 6 \end{vmatrix}} \quad y = \frac{\begin{vmatrix} 4 & 2 & 4 \\ 2 & 2 & 5 \\ 7 & 5 & 6 \end{vmatrix}}{\begin{vmatrix} \\ \\ \end{vmatrix}} \quad z = \frac{\begin{vmatrix} 4 & -1 & 2 \\ 2 & 3 & 2 \\ 7 & -2 & 5 \end{vmatrix}}{\begin{vmatrix} \\ \\ \end{vmatrix}}$$

$x = \frac{-33}{-11} = 3,\ y =$

or Solve by defs: $5x - y + 4z = 5,\ 2x + 3y + 5z = 2,\ 7x - 2y + 6z = 5$ (Ans 3, 2, −2)

(e) $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \cdot \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$ → Too much (!)

Product = $\begin{vmatrix} [a_1b_2c_3] & 0 & 0 \\ 0 & [a_1b_2c_3] & 0 \\ 0 & 0 & [a_1b_2c \end{vmatrix}$

since $A_1a_1 + A_2a_2 + A_3a_3 \ldots = [a_1b_2$

$\& A_1b_1 + A_2b_2 + \ldots =$

Too much!

(d) $\begin{vmatrix} x & y & z \\ z & x & y \\ y & z & x \end{vmatrix} \cdot \begin{vmatrix} a & c & b \\ b & a & c \\ c & b & a \end{vmatrix}$ → Ans is $\begin{vmatrix} X & Y & Z \\ Z & X & Y \\ Y & Z & X \end{vmatrix}$ where $X = ax + by + cz$, $Y = ay + bz + cx$, $Z = az + bx + cy$

(e) Shortcut $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc.$

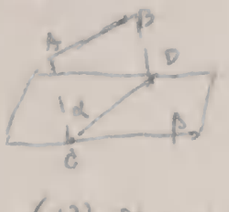
(f) $\left.\begin{aligned} x + 2y + 3z &= 6 \\ 2x + 4y + z &= 7 \\ 3x + 2y + 9z &= 14 \end{aligned}\right\}$ $x = \frac{\begin{vmatrix} 6 & 2 & 3 \\ 7 & 4 & 1 \\ 14 & 2 & 9 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 2 & 9 \end{vmatrix}}$; $y = \frac{\begin{vmatrix} 1 & 6 & 3 \\ 2 & 7 & 1 \\ 3 & 14 & 9 \end{vmatrix}}{\begin{vmatrix} \\ \\ \end{vmatrix}}$, $z = \frac{\begin{vmatrix} 1 & 2 & 6 \\ 2 & 4 & 7 \\ 3 & 2 & 14 \end{vmatrix}}{\begin{vmatrix} \\ \\ \end{vmatrix}}$

each determinant = −20 (!) so that $x = y = z = 1$

Solid Geometry: Hart & Hart

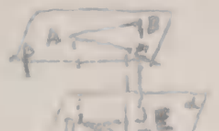
(1) one and only one ⊥ can be constructed to a plane from a point outside it

✓(2) If a line outside a plane is ∥ to a line of the plane, it is ∥ to the plane



AB & CD lie in a plane α. If AB were to intersect β at a point X, then X would be in γ and also in β & hence in the int^n of γ and β i.e. in CD i.e. AB would meet, which is impossible

✓(3) If each of two intersecting lines is ∥ to a plane, their plane is ∥ to the given plane



Draw AD ⊥ to α. Plane CAD cuts α in DF ∥ to CD. Since DE ∥ to AB.

AD ⊥ DF and AD ⊥ DE ∴ AD ⊥ AC & AD ⊥ AB ∴ AD ⊥ β and ∥ to

(7) Find what number must be added to 20, 8, 2 so that the resulting nos. will be in G.P.

[Let x = required no. Then $20+x$, $8+x$, $2+x$ are in G.P

$$\frac{8+x}{20+x} = \frac{2+x}{8+x} \quad \text{i.e. } (8+x)^2 = (2+x)(20+x) \longrightarrow x = 4.$$

Q.2

(8) Prove, with the usual notation, that ${}^nC_r = {}^nC_{n-r}$, and find n if value of $(n+1)C_{n-1} = 36$

[${}^{n+1}C_{n-1} = {}^{n+1}C_2 = \frac{(n+1)n}{1\cdot 2} = 36$ i.e. $n^2+n-72=0$ i.e. $n = \frac{-1 \pm \sqrt{1+288}}{2}$

$= \frac{-1 \pm \sqrt{288}}{2} = \frac{-1 \pm 17}{2}$

i.e. $n = 8$]

(9) Prove the binomial theorem for any integer a positive integral index, and prove that

$(1+x)^n = 1 + C_1 x + \cdots + C_n x^n$

${}^nC_1 + 2{}^nC_2 + 3{}^nC_3 + \cdots + n{}^nC_n = n \cdot 2^{n-1}$.

[$C_1 + 2C_2 + \cdots + nC_n = n + \frac{2n(n-1)}{1\cdot 2} + \frac{3n(n-1)(n-2)}{1\cdot 2\cdot 3} + \cdots + n$

$= n\left\{1 + (n-1) + \frac{(n-1)(n-2)}{1\cdot 2} + \cdots + 1\right\} = n(1+1)^{n-1} = n \cdot 2^{n-1}$

Q.3

(10) State the binomial theorem for a rational index, and show that, up to a first approximation $\sqrt{\frac{1+3x}{1-3x}} = 1+3x$.

(11) one on partial fractions $\longrightarrow$ find coeff of x^n in expn of $\frac{1}{1-5x+6x^2}$.

$\frac{1}{1-5x+6x^2} = \frac{3}{1-3x} - \frac{2}{1-2x}$

$= 3\{1+3x+(3x)^2 + \cdots + (3x)^n + \cdots\}$

$-2\{1+2x + \cdots + (2x)^n + \cdots\}$

required coeff $= 3^{n+1} - 2^{n+1}$

(12) Q.1. on logarithms.

Q.4 – Logarithmic & exponential fns.

(a) Prove that $\sum_{n=1}^{\infty} n^3/n! = 5e$ (Chrystal, p. 235

$n^3 \equiv n + 3n(n-1) + n(n-1)(n-2)$

$\therefore \sum \frac{n^3}{n!} = \sum \frac{1}{(n-1)!} + 3\sum \frac{1}{(n-2)!} + \sum \frac{1}{(n-3)!} \equiv 5e.$

(b)

Q.5 – determinants & linear eqns.

(a) Find a value of x that makes $\begin{vmatrix} 5+x & 7 & 3 \\ 6-x & 4 & 1 \\ 11 & 11 & 8 \end{vmatrix} = 0$

or $\begin{vmatrix} 3 & 8+x & 4 \\ 2 & 7 & 8-x \\ 1 & 4 & -1 \end{vmatrix} = 0$. or $\begin{vmatrix} 3+x & 5 & 2 \\ 1 & 7+x & 6 \\ 2 & 5 & 3+x \end{vmatrix}$ $\longrightarrow$ Ans is $x = 0, -1, -12$

(b) Find the product of the two determinants $\begin{vmatrix} & & \\ & & \\ & & \end{vmatrix} \times \begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}$

(c) Determine k so that the given system of eqns will have solns for which the unknowns are not all $= 0$.

$kx - 2y + kz = 0$, $3x - 4y + z = 0$, $2x + y + 3z = 0$ (Ans $k = 7$.

(2) A normal matrix is nilpotent if and only if it is the zero matrix - True - Proof - p. 243

p. 191 - (1) Prove that the rank of a (real) skew-symmetric matrix is an even integer - Proof on p. 248

(2)

28/8/68 — Q's for Students Submitting Papers.

(A) Algebra & Trigonometry.

Q.1 (1) Find the value of $\frac{16.08}{47 \times 0.0158}$ using logarithmic tables [Teach's Intermediate Algebra p.15

Let x = given expn, $\log x = \log 16.08 - \log 47 - \log 0.0158$

$= 1.2062 - 1.6721 - \overline{2}.1987 = 1.2062 - 1.6721 + 2 - 0.1987$

$= 3.2062 - 1.8708 = 1.3354$

$\therefore x = 21.65$.

(2) The sum of three numbers in A.P is 21, and the sum of their squares is 165. Find the numbers.

Let $a-d, a, a+d$ be the nos. The sum $= 3a = 21$ i.e. $a = 7$

$(a-d)^2 + a^2 + (a+d)^2 = 165$ i.e. $3a^2 + 2d^2 = 165$ i.e. $2d^2 = 165 - 147 = 18$

i.e. $d^2 = 9$ i.e. $d = \pm 3$ & the nos. are 4, 7, 10.

✓(3) Show that the sum to n terms of an A.P. whose first term is a & common difference is d is given by $S = \frac{n}{2}\{2a + (n-1)d\}$ & hence find the A.P ~~whose~~ in which the sum to n terms is $4n^2 + 7n$

[1st Part-Bkwork - 2nd part: $4n^2 + 7n = \frac{n}{2}(8n + 14) = \frac{n}{2}\{2(11) + (n-1)8\}$

$\therefore a = 11, d = 8$.

✓(4) Define the A.M & G.M of two positive real numbers a & b, and show that the A.M is always $\geq$ the G.M

$(\sqrt{a} - \sqrt{b})^2 \geq 0$, $a + b - 2\sqrt{ab} \geq 0$ i.e. $a + b \geq 2\sqrt{ab}$ i.e. $\frac{a+b}{2} \geq \sqrt{ab}$

~~(5) The sum of the first three terms of a G.P is 7, and the sum of the first four terms is 15. Find the G~~

$a\frac{(1-r^3)}{1-r} = 7$, $\frac{a(1-r^4)}{1-r} = 21$ Δ $\frac{1-r^4}{1-r^3} = 7$ i.e. $\frac{(1+r)(1+r^2)}{1+r+r^2} = 7$

$1 + r + r^2 + r^3 = 7(1 + r + r^2)$ $r^2 - 6r^2 - 6r - 6 = 0$.

~~(6) The sum of the first three terms of a G.P is 7, and sum of first six terms = 63. Find the G.P.~~

$a + ar + ar^2 = 7$, $a + ar + ar^2 + ar^3 + ar^4 + ar^5 = 63$ $ar^3(1 + r + r^2 + r^3) = 56$

Let $A = S^{-1}BS$. Then $\det(\lambda I_n - A) = \det(\lambda I_n - S^{-1}BS)$

$= \det\{S^{-1}(\lambda I_n - B)S\} = \det(S^{-1})\det(\lambda I_n - B)\det(S)$

$= \{\det(S)\}^{-1}\det(\lambda I_n - B)\det(S) = \det(\lambda I_n - B)$

Converse not true for eg $\begin{pmatrix}1&1\\0&1\end{pmatrix}$ and $I_2 = \begin{pmatrix}1&0\\0&1\end{pmatrix}$ have same characteristic polynomial $(\lambda-1)^2$.

However, for all non-singular 2-square matrices S, $S^{-1}I_2S = S \neq \begin{pmatrix}1&1\\0&1\end{pmatrix}$ for any S ie no similarity

p.153 (d) Two matrices are similar if & only if they have the same characteristic roots. } trivial

False : $\begin{pmatrix}0&0\\0&0\end{pmatrix}$ & $\begin{pmatrix}0&1\\0&0\end{pmatrix}$ have same ch. roots but are not similar }

(e) matrices $\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix}$ & $\begin{pmatrix}1&1&1\\0&2&1\\0&0&3\end{pmatrix}$ have same characteristic roots

True: they have the same ch. poly. $(\lambda-1)(\lambda-2)(\lambda-3)$ Qed

p.158 - Theorem: Every real matrix with real characteristic roots is orthogonally similar to a real triangular matrix

p.165: Show that the ch. roots of $A = \begin{pmatrix}1&0&-2\\-1&1&0\\0&-2&4\end{pmatrix}$ are all real. Find an orthogonal matrix P such that P^TAP is Δ^r.

Proof on p.239: ch. roots are $0,3,3$ and with $P = \frac{1}{3}\begin{pmatrix}2&2&1\\2&-1&-2\\1&-2&2\end{pmatrix}$

$$P^TAP = \begin{pmatrix}0&0&0\\0&3&-3\\0&0&3\end{pmatrix}.$$

p.168: Theorem: (i) A normal matrix is hermitian if & only if its ch. roots are real

(ii) " skew-hermitian " " pure imaginary

(iii) " unitary " " are of modulus 1.

p.173. Theorem: If A & B be n-sq. normal matrices, then $AB = BA$ if & only if there exists an n-sq. unitary matrix U such that U^*AU & U^*BU are both diagonal.

p.175 (1) The matrix $\begin{pmatrix}1+i&i\\i&1-i\end{pmatrix}$ is normal.

True - conj. transpose is $\begin{pmatrix}1-i&-i\\-i&1+i\end{pmatrix}$ and $\begin{pmatrix}1+i&i\\i&1-i\end{pmatrix}\begin{pmatrix}1-i&i\\-i&1+i\end{pmatrix} = \begin{pmatrix}3&0\\0&3\end{pmatrix}$

$= \begin{pmatrix}1-i&-i\\-i&1+i\end{pmatrix}\begin{pmatrix}1+i&i\\i&1-i\end{pmatrix}$

Worked Examples from F. M. Hall's Intro to Abstract Algebra Vol I.

(1) On the set of integers are the following equivalence relations? } Proof on p.21

(a) $a R b$ means that $a - b$ is even

(b) " " " odd

(2) Exc. 1, p. 46. (Set Theory) – Find complement of $(A \cup \bar{B}) \cap (\bar{A} \cup B \cup C)$ and write down the dual result.

(3) Exc. 2, p. 101 (Residue classes) – Solve $39x \equiv 17 \ (67)$

(4) Exc. 3, p. 125 (Polynomials) – Factorise $B(x) \equiv x^6 + 1$ into irreducible factors over (a) the Complex nos, (b) the real nos. (c) the rationals, and (d) the integers.

Also worked examples from Marcus & Minc's – Intro to linear algebra.

p.63 (1) The product of two hermitian matrices is hermitian – false – take $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $B \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

then $AB = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is not hermitian. If, however, A and B were to commute, then

$(AB)^* = B^* A^* = BA = AB$ and it would follow that AB is hermitian

(2) If the product of two hermitian matrices is 0, then one of them must be 0 – false, take

$A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

(3) If A is a hermitian matrix and $A^2 = 0$, then $A = 0$ – True, $0 = A^2 = AA^*$. Hence

$0 = (AA^*)_{ii} = \sum_{j=1}^{n} a_{ij} \bar{a}_{ij} = \sum_{j=1}^{n} |a_{ij}|^2$, $(i = 1, 2, \cdots n)$. It follows that $a_{ij} = 0$

$i, j = 1, \cdots n$.

(4) If A and B be n-rowed hermitian matrices & $A^2 + B^2 = 0$, then $A = B = 0$ – True as above

p.64 (1) Show that the product of two unitary matrices is unitary

Proof: If A & B be unitary, then $(AB)^*(AB) = B^*(A^*A)B = B^*B = I_n$. Similarly

$(AB)(AB)^* = I_n$.

p.74. If R be the real [illegible], then $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$ are similar matrices

Proof :- If $C = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$, then $C^{-1} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} C = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$

p.148 Theorem: Similar matrices have the same characteristic polynomial

Proof :- P. T. O ←

Alladi Ramakrishnan: Article on "Topological features of Feynman graphs" in J. Math. Analysis and its applications, p. 68

Also "Graphical representation of CPT, p. 147

[Along with S. Chandrasekhar, he is also an associate Editor of this Journal]

Book by him on "Elementary Particles and Cosmic rays" Macmillan, Pergamon Press, New York, 1962.

$(2\alpha^2-\beta^2)(2\gamma^2-\delta^2) = (2\alpha\gamma+\beta\delta)^2 - 2(\beta\gamma+\alpha\delta)^2$

$x^2-2y^2 = 2(x+y)^2-(x+2y)^2.$

} Every prime $8n\pm1$ is represented by $2t^2-u^2$ and by u^2-2t^2.

Dirichlet's discussion of biquadratic character of 2 modulo p.

$$p = 8n+1 = t^2+2u^2 = \phi^2+\psi^2$$

where p is prime & ψ may be taken to be a multiple of 4. If m is the g.c.d of $\phi = m\phi'$ and $u = mu'$, it is shown $t+\psi = EK$, $t-\psi = FL$, $m^2 = EF$, $\phi'^2-2u'^2 = KL$,

where E & F are relatively prime, whence E is a square & $E \equiv 1 \pmod 8$. Since K is an odd divisor of $\phi'^2-2u'^2$, $K \equiv 1$ or 7, $t+\psi \equiv 1$ or 7. Thus if $\psi = 0$, $t \equiv 1$ or 7, but if $\psi = 4$, $t \equiv 3$ or $5 \pmod 8$.

Jacobi noted that the above theorem of Dirichlet is equivalent to the following:- If a prime $8h+1$ is represented by two forms $f \equiv (4m+1)^2+16n^2$, $g = (4m'+1)^2+8n'^2$, then $m+n$ and n' are both even or both odd.

$$(a+b+1)^2+(a-b)^2 = 4\left\{\frac{a^2+a}{2}+\frac{b^2+b}{2}\right\}+1.$$

$4mn-m-n$ is never a square of or a triangular number

[never a square because $a^2+1 = (4m-1)(4n-1)$ is impossible]

---

30 X 72

27 X 72

$\lim_{|x|\to\infty} \dfrac{u_c(x)}{\left(\frac{1}{x}\right)^p \log^q\left(\frac{1}{x}\right)} = \text{const} \neq 0.$

(S. R. Srinivasa - J. I. Sc.)

(S. D. Dwarkanath)

Rack - 89 : Upstairs - Ground Floor } → Sc. Amer.

Rack - 63 - Downstairs - " → Phys. Rev. 01/01 & 01/9, 01/8, 7/10

Rack - 52 - " - " → Nature. 2/10/40

" - 53 - " - " → Proc. Roy. Soc. 1/10/40

" - 62 - " " → Rev. Mod. Phys.

" - 96 - Upstairs - " → Acta. Phys. Pol.

Proc. Nat. Acad. Sci (U.S.A) --- 89. (Ground floor upstairs)

Progress. Th. Physics (Japan) — 93 (-- do --)

Annales d'Inst. H. Poincaré — 92.

" de Physique — 93

Bull. Soc. Math. France — 92

Agammaglobulinemia

This non-sup word refers to a disease in which the blood lacks gamma globulin. Because anti-bodies are found in gamma globulin, those who suffer from this disease are prey to grave infections.